Figure 7.1 **<u>Typical .dfg template input file for testing axiomatic translation</u>** for the problem `bp,ddnp,g.dfg` $(\wedge(\wedge(\Box(r,p),\Diamond(r,\Diamond(r,\neg p))),\wedge(\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,q_1)))))),\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,\Diamond(r,q_2))))))))).$ Note in particular, the list of settings for extended-SPASS. This file is suitable for local satisfiability calculations (as seen in figure 7.5-7.20) in extended-SPASS version 1.1.0, and for global satisfiability calculations (see figure 7.21-7.25) in extended-SPASS version 1.1.2. In the latter, local satisfiability calculations require the user to move the target formula to the conjectures list, and to negate it.

```
begin_problem(zzzzzz).
list_of_descriptions.
  name({* Problem:KK-bp,ddnp,g for extended-SPASS version 1.1.0 *}).
  author({* *}).
  status(unknown).
  description({*
translate([(K,r)],and(and(box(r,p),dia(r,dia(r,not(p)))),and(dia(r,dia(r,dia(r
,dia(r,dia(r,dia(r,q1)))))),dia(r,dia(r,dia(r,dia(r,dia(r,dia(r,q2)))))))).
*}).
end_of_list.

list_of_symbols.
predicates[(R,2),(r,0),(p,0),(q1,0),(q2,0)].
end_of_list.

list_of_special_formulae(axioms,eml).
prop_formula(
and(and(box(r,p),dia(r,dia(r,not(p)))),and(dia(r,dia(r,dia(r,dia(r,dia(r,dia(r
,q1)))))),dia(r,dia(r,dia(r,dia(r,dia(r,dia(r,q2)))))))
).
end_of_list.

list_of_special_formulae(conjectures,EML).
end_of_list.

list_of_settings(SPASS).
{*
set_flag(EMLAxiom,1).
set_flag(DocProof,0).
set_flag(Sorts,0).
set_flag(CNFRenaming,0).
set_flag(CNFOptSkolem,0).
set_flag(CNFStrSkolem,0).
set_flag(Select,0).
set_flag(Splits,-1).
set_flag(PGiven,0).
set_flag(PProblem,0).
*}
end_of_list.
```

Figure 7.2 **<u>Typical script used to execute a series of modal-axiom combinations</u>** with a series of .dfg files, one of which is `bp,ddnp,g.dfg` (as seen in figure 7.1).

```
#!/bin/csh
setenv t 200
cd template
foreach f ( *.dfg )
 nice SPASS -TimeLimit=$t -EMLAxiomLogT=1    $f > ../out/`basename $f .dfg`.T.out
 nice SPASS -TimeLimit=$t -EMLAxiomLogD=1    $f > ../out/`basename $f .dfg`.D.out
 nice SPASS -TimeLimit=$t                    $f > ../out/`basename $f .dfg`.K.out
 nice SPASS -TimeLimit=$t -EMLAxiomLogAlt1KK=1 -EMLAxiomLogFac1=1 -EMLAxiomLogFac2=2
                                    $f > ../out/`basename $f  .dfg`.alt1^1,2.out
…etc…
end
```

Figure 7.3 **Typical script and .dfg file** used to execute a particular problem and modal-axiom combination, when using the `set_axiom` syntax to specify the axiom used. This example uses formula `bp,ddnp,g` in axiom TR_o. Note in particular, the list of settings for extended-SPASS.

```
begin_problem(zzzzzz).
list_of_descriptions.
  name({* Problem:KK-bp,ddnp,g for extended-SPASS version 1.1.0  *}).
  author({* *}).
  status(unknown).
  description({*
translate([(K,r)],and(and(box(r,p),dia(r,dia(r,not(p)))),and(dia(r,dia(r,dia(r,dia(r,dia
(r,dia(r,q1)))))),dia(r,dia(r,dia(r,dia(r,dia(r,dia(r,q2)))))))). *}).
end_of_list.

list_of_symbols.
predicates[(R,2),(r,0),(p,0),(q1,0),(q2,0)].
end_of_list.

list_of_special_formulae(axioms,eml).
prop_formula(
and(and(box(r,p),dia(r,dia(r,not(p)))),and(dia(r,dia(r,dia(r,dia(r,dia(r,dia(r,q1)))))),
dia(r,dia(r,dia(r,dia(r,dia(r,dia(r,q2)))))))
).
end_of_list.

list_of_special_formulae(conjectures,EML).
end_of_list.

list_of_settings(SPASS).
{*
set_flag(EMLAxiom,1).
set_flag(DocProof,0).
set_flag(Sorts,0).
set_flag(CNFRenaming,0).
set_flag(CNFOptSkolem,0).
set_flag(CNFStrSkolem,0).
set_flag(Select,0).
set_flag(Splits,-1).
set_flag(PGiven,0).
set_flag(PProblem,0).
set_axiom([r,TRo]).
*}
end_of_list.

end_problem.
```

```
#!/bin/csh

setenv t 200
cd template
foreach f ( *.dfg )
 echo -n "========="; echo $f
      nice SPASS -TimeLimit=$t $f > ../out/`basename $f .dfg`.out
end
exit
```

Figure 7.4 **Formulae used for software testing:** This is the *test formula set*. Each target formula is numbered, and given a descriptive name. These are both used elsewhere to refer to specific formula from this test set. Formulae 1-119 were used in the source code of program `ml2dfg` [mentioned in 1]. Formulae 120-133 are used to assess the special cases for handling true/false. The formulae marked * were chosen for implementation as test cases in Jasper.

No.	Descriptive Name	Target Formulae
1*	4	¬(→(□p,□□p))
2*	4^2	¬(→(□p,□□□p))
3*	4^3	¬(→(□p,□□□□p))

4*	5	¬(→(¬□¬□p,□p))
5*	5^2	¬(→(¬□□¬□p,□p))
6*	5^3	¬(→(¬□□□¬□p,□p))
7*	D	¬(→(□p,◇p))
8*	T	¬(→(□p,p))
9*	alt1	¬(→(◇p,□p))
10	alt1^1,1	¬(→(◇◇p,□□p))
11	alt1^1,2	¬(→(◇◇p,□□□p))
12	alt1^2,1	¬(→(◇◇◇p,□□p))
13	alt1^2,2	¬(→(◇◇◇p,□□□p))
14*	B	¬(→(¬□¬□p,p))
15*	B^2	¬(→(¬□□¬□p,p))
16	F	¬(→(∧(□◇p,□◇q),◇(∧(p,q))))
17*	M	¬(→(□¬□¬p,¬□¬□p))
18*	Cxt	¬(→(¬□¬□p,□□p))
19*	bbp,dddd(q,dnp)	∧(□□p,◇◇◇◇(∧(q,◇¬p)))
20*	bp,dddnp,g	∧(∧(□p,◇◇◇¬p),◇◇◇◇◇◇q)
21	bp,dddp,g	∧(∧(□p,◇◇◇p),◇◇◇◇◇◇q)
22*	bp,ddnp,g	∧(∧(□p,◇◇¬p),∧(◇◇◇◇◇◇q_1,◇◇◇◇◇◇q_2))
23	bp,ddp	∧(□p,◇◇p)
24	bq,d	∧(□p,◇⊤)
25*	bq,dddddddnq	∧(□p,◇◇◇◇◇◇◇¬p))
26*	bq,dddddnq	∧(□p,◇◇◇◇◇¬p)
27	d(bp,ddnq),ddbq	∧(◇(∧(□p,◇◇¬q)),◇◇□q)
28	d(dddddnq,ddddbq)	◇(∧(◇◇◇◇◇¬q,◇◇◇◇□q))
29	d(ddddnq,dddbq)	◇(∧(◇◇◇◇¬q,◇◇◇□q))
30	d(ddnq,dbq)	◇(∧(◇◇¬q,◇□q))
31	d(dnq,dbq)	◇(∧(◇¬q,◇□q))
32	d(dnq,dddbq)	◇(∧(◇¬q,◇◇◇□q))
33	dbp,dd(np,bq)	∧(◇□p,◇◇(∧(¬p,□q)))
34	dbp,dddddnp	∧(◇□p,◇◇◇◇◇¬p)
35	dbp,ddnp	∧(◇□p,◇◇¬p)
36	dbq,d(dnq,ddp)	∧(◇(∧(◇¬q,◇◇p)),◇□q)
37	dd(bq,ddddnq)	◇◇(∧(□p,◇◇◇◇¬p))
38	dd(dnq,dbq)	◇◇(∧(◇¬q,◇□q))
39*	ddddddnq,dp,bbp	∧(∧(◇◇◇◇◇◇¬q,◇p),□□q)
40*	dddddnq,bbbq	∧(◇◇◇◇◇¬q,□□□q)
41	dddddnq,ddbq	∧(◇◇◇◇◇¬q,◇◇□q)
42	dddddnq,dddbq	∧(◇◇◇◇◇¬q,◇◇◇□q)
43	dddddnq,ddddbq	∧(◇◇◇◇◇¬q,◇◇◇◇□q)
44*	ddddnq,b(q,g)	∧(◇◇◇◇¬q,□(∧(p,q)))
45*	ddddnq,bbp	∧(◇◇◇◇¬q,□□q)
46*	ddddnq,bq	∧(◇◇◇◇¬q,□q)
47	ddddnq,ddbq	∧(◇◇◇◇¬q,◇◇□q)
48	ddddnq,dddbq	∧(◇◇◇◇¬q,◇◇◇□q)
49*	ddddnq,dp,bbp	∧(∧(◇◇◇◇¬q,◇p),□□q)
50*	dddnq,bbq	∧(◇◇◇¬q,□□q)
51*	dddnq,bq	∧(◇◇◇¬q,□q)
52	dddnq,dbq	∧(◇◇◇¬q,◇□q)
53	dddnq,ddbq	∧(◇◇◇¬q,◇◇□q)
54	dddnq,dddbq	∧(◇◇◇¬q,◇◇◇□q)
55	dddnq,ddddbq	∧(◇◇◇¬q,◇◇◇◇□q)
56*	dddnq,dp,bbp	∧(∧(◇◇◇¬q,◇p),□□q)
57*	ddnq,bq	∧(◇◇¬q,□q)
58	ddnq,dbq	∧(◇◇¬q,◇□q)
59	ddnq,ddbq	∧(◇◇¬q,◇◇□q)
60	ddnq,dddbq	∧(◇◇¬q,◇◇◇□q)
61	ddnq,dp,bbp	∧(∧(◇◇¬q,◇p),□□q)
62	Nnd	¬(¬◇⊤)
63	dnq,bq	∧(◇¬q,□q)
64*	dnq,dbq	∧(◇¬q,◇□q)
65	dnq,ddbq	∧(◇¬q,◇◇□q)
66	dnq,dddbq	∧(◇¬q,◇◇◇□q)

67	dnq,ddp,bbbq	∧(∧(◇¬q,◇◇p),□□□q)
68	n(bp,dp)	¬(→(□p,◇p))
69	n(d1d2b3bp->p)	¬(→(◇(∧(p_1,◇(∧(p_2,□(∧(p_3,□p)))))),p))
70	n(dbbbbp->bbdp)	¬(→(◇□□□□p,□□◇p))
71	n(dbbp->bp)	¬(→(◇□□p,□p))
72	n(dbdnp->nbbp)	¬(→(◇□◇¬p,¬□□p))
73	n(dbp->dp)	¬(→(◇□p,◇p))
74	n(ddbbp->p)	¬(→(◇◇□□p,p))
75	n(dp->p)	¬(→(◇p,p))
76	nd,g	¬(◇(∨(p,¬p)))
77	nd	¬◇⊤
78	np,bbbp,dddddddq	∧(∧(¬p,□□□p),◇◇◇◇◇◇◇◇q)
79	np,bbp,ddddddddq	∧(∧(¬p,□□p),◇◇◇◇◇◇◇◇q)
80	np,ddddddddbp	∧(¬p,◇◇◇◇◇◇◇◇□p)
81	np,dddddddddbp	∧(¬p,◇◇◇◇◇◇◇◇□q)
82	nq,ddbq	∧(¬q,◇◇□q)
83	nq,dddddbq	∧(¬p,◇◇◇◇◇□p)
84	p,bbndp,ddq	∧(p,∧(□(□(¬(◇p))),◇(◇q)))
85	(db)^1p,p	¬(→(◇□p,p))
86	(db)^1p,q	¬(→(◇□p,q))
87	(db)^2p,p	¬(→(◇□◇□p,p))
88	(db)^2p,q	¬(→(◇□◇□p,q))
89	(db)^3p,p	¬(→(◇□◇□◇□p,p))
90	(db)^3p,q	¬(→(◇□◇□◇□p,q))
91	(db)^4p,p	¬(→(◇□◇□◇□◇□p,p))
92	(db)^4p,q	¬(→(◇□◇□◇□◇□p,q))
93	(db)^5p,p	¬(→(◇□◇□◇□◇□◇□p,p))
94	(db)^5p,q	¬(→(◇□◇□◇□◇□◇□p,q))
95	(db)^6p,p	¬(→(◇□◇□◇□◇□◇□◇□p,p))
96	(db)^6p,q	¬(→(◇□◇□◇□◇□◇□◇□p,q))
97	(db)^7p,p	¬(→(◇□◇□◇□◇□◇□◇□◇□p,p))
98	(db)^7p,q	¬(→(◇□◇□◇□◇□◇□◇□◇□p,q))
99	(db)^8p,p	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□p,p))
100	(db)^8p,q	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□p,q))
101	(db)^9p,p	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□◇□p,p))
102	(db)^9p,q	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□◇□p,q))
103	(db)^10p,p	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□◇□◇□p,p))
104	(db)^10p,q	¬(→(◇□◇□◇□◇□◇□◇□◇□◇□◇□◇□p,q))
105	aiml02_prop3i	¬(→(□(→(□p,□q)),□□(→(□p,q))))
106	aiml02_prop3ii	¬(→(□□(→(□p,q)),□(→(□p,□q))))
107*	aiml02_prop3iii	¬(→(◇(□(→(□p,q))),□(→(□p,◇q))))
108*	amai02	¬(→(□(→(□p,q)),□(→(□p,□(→(□p,q))))))
109*	amai02b	¬(→(¬(□(→(□p,q))),□(→(□p,¬(□(→(□p,q)))))))
110*	demri1	¬(↔(□(∨(□p,□q)),∨(□p,□q)))
111*	demri2	¬(→(□(→(□(↔(p,q)),t)),□(→(□(↔(p,q)),□⊤))))
112	demri3	¬(↔(□p,∧(□(→(q,p)),□(→(¬q,p)))))
113	demri5	¬(◇(□(↔(□(∨(p,□q)),∨(□p,□q)))))
114	demri6	¬(◇(□(↔(→(p,q),∨(¬q,∨(∧(∨(¬p,∨(∧(q,◇(∧(p,¬q))), ¬(◇(∧(p,q))))),◇(∧(q,¬(∨(¬p,∨(∧(q,◇(∧(p,¬q))),¬◇(∧(p,q)))))))), ¬(◇(∧(q,∨(¬p,∨(∧(q,◇(∧(p,¬q))),¬◇(∧(p,q))))))))))))
115	demri7	¬(□(→(□(→(□p,□(→(□q,□⊤)))),□(→(□(→(□p,□q)),□(→(□p,□⊤)))))))
116	demri8	¬(□(→(□(→(□p,□(→(□q,□⊤)))),□(→(□q,□(→(□p,□⊤)))))))
117	demri9	¬(□(→(□(→(□p,□(→(□q,□⊤)))),□(→(□(→(□p,□q)),□⊤)))))
118*	CR	¬(→(◇(r,□(a,p)),□(a,◇(r,p))))
119	demri4	¬(→(∧(□(r_c,→(¬p_c,□(r_b,¬p_c))),∧(□(r_c,□(r_b,□(r_a,∨(p_a,∨(p_b,p_c))))), ∧(□(r_c,□(r_b,→(¬p_b,□(r_a,¬p_b)))),∧(□(r_c,□(r_b,→(¬p_c,□(r_a,¬p_c)))), ∧(□(r_c,¬(□(r_b,p_b))),□(r_c,□(r_0,¬(□(r_a,p_a)))))))))),□(r_c,p_c)))
120	bbf	□□⊥
121	bbt	□□⊤
122	bdf	□◇⊥
123	bdt	□◇⊤
124	bf	□⊥

125	bt	$\Box\top$
126	dbf	$\Diamond\Box\bot$
127	dbt	$\Diamond\Box\top$
128	ddf	$\Diamond\Diamond\bot$
129	ddt	$\Diamond\Diamond\top$
130	df	$\Diamond\bot$
131	dt	$\Diamond\top$
132	fa	$\bot$ or $\neg\top$
133	tr	$\top$ or $\neg\bot$

Figure 7.5 <u>Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed single axioms</u>. The test set of formulae is listed in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. This data is the composite of two sets of experiments. Between the sets of experiments only the number of unsolved experiments changed. Where an experiment produced a result (Proof or Completion) in both sets, it was always the same. For experiments that varied between the two sets of data, the result (either P or C) is reported, not the failure (X). The total execution time for the data in this table is 22.2 hours (22.2 hours*2). The data was extracted from saved output files using a C-shell script, and processed in Microsoft Excel.

No.	Target Formula	K	T	B	D	4	5_o	alt_1	4^2	4^3	5^2	5^3	alt_1^{11}	alt_1^{21}	alt_1^{12}	alt_1^{22}
1	4	C	C	C	C	P	C	C	C	C	C	X	C	C	C	C
2	4^2	C	C	C	C	P	C	C	P	C	C	X	C	C	C	X
3	4^3	C	C	C	C	P	C	C	C	P	C	X	C	C	X	X
4	5	C	C	C	C	C	P	C	C	C	C	X	C	C	C	C
5	5^2	C	C	C	C	C	P	C	C	C	P	X	C	C	C	X
6	5^3	C	C	C	C	C	P	C	C	C	C	P	C	C	C	X
7	D	C	P	C	P	C	C	C	C	C	C	X	C	C	C	C
8	T	C	P	C	C	C	C	C	C	C	C	C	C	C	C	C
9	alt1	C	C	C	C	C	C	P	C	C	C	X	C	C	C	C
10	alt1^1,1	C	C	C	C	C	C	P	C	C	C	X	P	C	C	X
11	alt1^1,2	C	C	C	C	C	C	C	C	C	X	X	C	P	P	X
12	alt1^2,1	C	C	C	C	C	C	C	C	C	X	X	C	P	P	X
13	alt1^2,2	C	C	C	C	C	C	P	C	C	X	X	P	P	P	P
14	B	C	C	P	C	C	C	C	C	C	C	X	C	C	C	C
15	B^2	C	C	C	C	C	C	C	C	C	C	X	C	C	C	X
16	F	C	C	C	C	C	C	C	C	C	C	X	C	X	X	X
17	M	C	C	C	C	C	C	C	C	C	C	X	C	C	C	X
18	Cxt	C	C	C	C	C	P	P	C	C	P	X	C	C	C	X
19	bbp,dddd(q,dnp)	C	C	C	C	P	P	C	C	P	X	X	C	P	X	X
20	bp,dddnp,g	C	C	C	C	P	C	C	P	C	X	X	X	X	X	X
21	bp,dddp,g	C	C	C	C	C	C	C	C	C	X	X	X	X	X	X
22	bp,ddnp,g	C	C	C	C	P	C	C	C	X	X	X	X	X	X	X
23	bp,ddp	C	C	C	C	C	C	C	C	C	C	X	C	C	C	C
24	bq,d	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
25	bq,ddddddddnq	C	C	C	C	P	C	C	P	P	X	X	C	X	X	X
26	bq,dddddnq	C	C	C	C	P	C	C	P	C	C	X	C	C	X	X
27	d(bp,ddnq),ddbq	C	C	C	C	C	P	P	C	C	P	P	P	P	P	X
28	d(dddddnq,ddddbq)	C	C	C	C	C	P	P	C	X	P	P	P	P	P	P
29	d(ddddnq,dddbq)	C	C	C	C	C	P	P	C	X	P	P	P	P	P	P
30	d(ddnq,dbq)	C	C	C	C	C	P	P	C	C	P	P	P	P	P	X
31	d(dnq,dbq)	C	C	C	C	C	P	C	C	C	C	X	C	C	C	X
32	d(dnq,dddbq)	C	C	C	C	C	P	C	C	C	X	P	C	C	P	X
33	dbp,dd(np,bq)	C	C	C	C	C	P	P	C	C	P	X	C	X	X	X
34	dbp,dddddnp	C	C	C	C	C	P	C	C	C	X	P	C	C	C	X
35	dbp,ddnp	C	C	C	C	C	P	P	C	C	P	X	C	C	C	X
36	dbq,d(dnq,ddp)	C	C	C	C	C	P	P	C	C	P	P	C	X	X	X
37	dd(bq,ddddnq)	C	C	C	C	P	P	C	C	P	X	P	C	P	P	X
38	dd(dnq,dbq)	C	C	C	C	C	P	C	C	C	C	X	C	P	P	X
39	ddddddnq,dp,bbp	C	C	C	C	P	P	C	P	C	P	X	C	P	X	X
40	dddddnq,bbbq	C	C	C	C	P	P	C	P	C	P	X	C	P	X	X
41	dddddnq,ddbq	C	C	C	C	C	P	C	C	X	P	X	C	P	P	X
42	dddddnq,dddbq	C	C	C	C	C	P	C	C	X	X	X	X	P	P	X
43	dddddnq,ddddbq	C	C	C	C	C	P	P	C	X	P	P	P	P	P	P
44	ddddnq,b(q,g)	C	C	C	C	P	C	C	C	P	X	X	C	C	X	X

45	ddddnq,bbp	C	C	C	C	P	P	C	P	C	P	X	C	P	X	X
46	ddddnq,bq	C	C	C	C	P	C	C	C	P	C	X	C	C	X	X
47	ddddnq,ddbq	C	C	C	C	C	P	C	C	C	X	X	C	P	P	X
48	ddddnq,dddbq	C	C	C	C	C	P	P	C	X	P	P	P	P	P	P
49	ddddnq,dp,bbp	C	C	C	C	P	P	C	P	C	P	X	C	P	X	X
50	dddnq,bbq	C	C	C	C	P	P	C	C	C	C	X	C	P	C	X
51	dddnq,bq	C	C	C	C	P	C	C	P	C	C	X	C	C	C	X
52	dddnq,dbq	C	C	C	C	C	P	C	C	C	C	X	C	C	C	X
53	dddnq,ddbq	C	C	C	C	C	P	P	C	C	P	P	P	P	P	X
54	dddnq,dddbq	C	C	C	C	C	P	C	C	C	X	X	C	P	P	X
55	dddnq,ddddbq	C	C	C	C	C	P	C	C	X	P	X	C	P	P	X
56	dddnq,dp,bbp	C	C	C	C	P	P	C	C	C	C	X	C	P	X	X
57	ddnq,bq	C	C	C	C	P	C	C	C	C	C	X	C	C	C	C
58	ddnq,dbq	C	C	C	C	C	P	P	C	C	P	X	C	C	C	X
59	ddnq,ddbq	C	C	C	C	C	P	C	C	C	C	X	C	C	C	X
60	ddnq,dddbq	C	C	C	C	C	P	C	C	C	P	X	C	C	P	X
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
62	Nnd	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
63	dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
64	dnq,dbq	C	C	C	C	C	P	C	C	C	C	X	C	C	C	C
65	dnq,ddbq	C	C	C	C	C	P	C	C	C	P	X	C	C	C	X
66	dnq,dddbq	C	C	C	C	C	P	C	C	C	C	P	C	C	C	X
67	dnq,ddp,bbbq	C	P	P	C	C	P	C	C	C	P	X	C	C	X	X
68	n(bp,dp)	C	P	C	P	C	C	C	C	C	C	X	C	C	C	C
69	n(d1d2b3bp->p)	C	C	P	C	C	C	C	C	C	X	X	C	C	X	X
70	n(dbbbbp->bbdp)	C	C	P	C	C	P	C	C	X	P	X	X	X	X	X
71	n(dbbp->bp)	C	C	P	C	C	P	C	C	C	C	X	C	C	C	X
72	n(dbdnp->nbbp)	C	P	C	C	C	P	C	C	C	C	X	C	C	C	X
73	n(dbp->dp)	C	P	C	C	C	P	C	C	C	C	X	C	C	C	C
74	n(ddbbp->p)	C	C	P	C	C	C	C	C	C	C	X	C	C	C	X
75	n(dp->p)	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
76	nd,g	C	P	C	P	C	C	C	C	C	C	X	C	C	C	C
77	nd	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
78	np,bbbp,dddddddq	C	P	C	C	C	C	C	C	C	X	X	X	X	X	X
79	np,bbp,dddddddq	C	P	P	C	C	C	C	C	C	X	X	X	X	X	X
80	np,ddddddddbp	C	C	C	C	C	C	C	C	C	X	X	X	X	X	X
81	np,dddddddddbp	C	C	C	C	C	C	C	C	C	X	X	X	X	X	X
82	nq,ddbq	C	C	C	C	C	C	C	C	C	C	X	C	C	C	X
83	nq,dddddbq	C	C	C	C	C	C	C	C	C	X	X	C	X	X	X
84	p,bbndp,ddq	C	P	C	C	C	C	C	C	C	X	X	C	C	X	X
85	(db)^1p,p	C	C	P	C	C	C	C	C	C	C	X	C	C	C	C
86	(db)^1p,q	C	C	C	C	C	C	C	C	C	C	X	C	C	C	C
87	(db)^2p,p	C	C	P	C	C	C	C	C	C	C	X	C	C	X	X
88	(db)^2p,q	C	C	C	C	C	C	C	C	C	C	X	C	C	X	X
89	(db)^3p,p	C	C	P	C	C	C	C	C	C	C	X	C	X	X	X
90	(db)^3p,q	C	C	C	C	C	C	C	C	C	C	X	C	X	X	X
91	(db)^4p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
92	(db)^4p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
93	(db)^5p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
94	(db)^5p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
95	(db)^6p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
96	(db)^6p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
97	(db)^7p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
98	(db)^7p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
99	(db)^8p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
100	(db)^8p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
101	(db)^9p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
102	(db)^9p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
103	(db)^10p,p	C	C	P	C	C	C	C	C	X	X	X	X	X	X	X
104	(db)^10p,q	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
105	aiml02_prop3i	C	C	C	C	C	P	C	C	C	X	X	C	C	X	X
106	aiml02_prop3ii	C	C	C	C	P	P	C	C	C	X	X	C	P	X	X
107	aiml02_prop3iii	C	C	C	C	C	P	C	C	C	X	X	C	X	X	X
108	amai02	C	C	C	C	P	C	C	C	C	C	X	C	C	C	X
109	amai02b	C	C	C	C	C	P	C	C	C	C	X	C	C	C	X
110	demri1	C	C	C	C	C	C	C	C	C	C	X	C	C	C	X
111	demri2	C	C	C	C	P	C	C	C	C	X	X	C	X	X	X
112	demri3	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
113	demri5	C	C	C	C	C	C	C	C	C	X	X	X	C	X	X
114	demri6	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
115	demri7	C	C	C	C	C	P	C	C	C	X	X	X	P	P	X
116	demri8	C	C	C	C	C	P	C	C	C	X	X	X	P	P	X
117	demri9	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
118	CR*	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
119	demri4*	P	P	P	P	P	P	P	P	P	P	P	X	X	X	X
120	bbf	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
121	bbt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
122	bdf	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
123	bdt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

124	bf	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
125	bt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
126	dbf	C	P	P	P	C	P	C	C	C	C	C	C	C	C	C
127	dbt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
128	ddf	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
129	ddt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
130	df	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
131	dt	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
132	fa	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
133	tr	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

*Axiom applied to all the modality indices in the input problem.

Figure 7.6 **Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations.** The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. Again, this data is a composite of two sets of experiments (see figure 7.5 for more details). The total execution time for the data in this table is 0.6 hours (0.6 hours*2).

No.	Target Formula	T4	TB	DB	D4	4_oB	5_oB	5_oT	$T4_oB$	D_o4_oB	T_c4B_c	D_cB	D_c4
1	4	P	C	C	P	P	P	P	P	P	P	C	P
2	4^2	P	C	C	P	P	P	P	P	P	P	C	P
3	4^3	P	C	C	P	P	P	P	P	P	P	C	P
4	5	C	C	C	C	P	P	P	P	P	P	C	C
5	5^2	C	C	C	C	P	P	P	P	P	P	C	C
6	5^3	C	C	C	C	P	P	P	P	P	P	C	C
7	D	P	P	P	P	C	C	P	P	P	P	P	P
8	T	P	P	C	C	C	C	P	P	P	P	C	C
9	alt1	C	C	C	C	C	C	C	C	C	C	C	C
10	alt1^1,1	C	C	C	C	C	C	C	C	C	C	C	C
11	alt1^1,2	C	C	C	C	C	C	C	C	C	C	C	C
12	alt1^2,1	C	C	C	C	C	C	C	C	C	C	C	C
13	alt1^2,2	C	C	C	C	C	C	C	C	C	C	C	C
14	B	C	P	P	C	P	P	P	P	P	P	P	C
15	B^2	C	C	C	C	P	P	P	P	P	P	C	C
16	F	C	C	C	C	C	C	C	C	C	C	C	C
17	M	C	C	C	C	C	C	C	C	C	C	C	C
18	Cxt	C	C	C	C	P	P	P	P	P	P	C	C
19	bbp,dddd(q,dnp)	P	C	C	P	P	P	P	P	P	P	C	P
20	bp,dddnp,g	P	C	C	P	P	P	P	P	P	P	C	P
21	bp,dddp,g	C	C	C	C	C	C	C	C	C	C	C	C
22	bp,ddnp,g	P	C	C	P	P	P	P	P	P	P	C	P
23	bp,ddp	C	C	C	C	C	C	C	C	C	C	C	C
24	bq,d	C	C	C	C	C	C	C	C	C	C	C	C
25	bq,dddddddnq	P	C	C	P	P	P	P	P	P	P	C	P
26	bq,dddddnq	P	C	C	P	P	P	P	P	P	P	C	P
27	d(bp,ddnq),ddbq	C	C	C	C	P	P	P	P	P	P	C	C
28	d(dddddnq,ddddbq)	C	C	C	C	P	P	P	P	P	P	C	C
29	d(ddddnq,dddbq)	C	C	C	C	P	P	P	P	P	P	C	C
30	d(ddnq,dbq)	C	C	C	C	P	P	P	P	P	P	C	C
31	d(dnq,dbq)	C	C	C	C	P	P	P	P	P	P	C	C
32	d(dnq,dddbq)	C	C	C	C	P	P	P	P	P	P	C	C
33	dbp,dd(np,bq)	C	C	C	C	P	P	P	P	P	P	C	C
34	dbp,dddddnp	C	C	C	C	P	P	P	P	P	P	C	C
35	dbp,ddnp	C	C	C	C	P	P	P	P	P	P	C	C
36	dbq,d(dnq,ddp)	C	C	C	C	P	P	P	P	P	P	C	C
37	dd(bq,ddddnq)	P	C	C	P	P	P	P	P	P	P	C	P
38	dd(dnq,dbq)	C	C	C	C	P	P	P	P	P	P	C	C
39	ddddddnq,dp,bbp	P	C	C	P	P	P	P	P	P	P	C	P
40	dddddnq,bbbq	P	C	C	P	P	P	P	P	P	P	C	P
41	dddddnq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
42	dddddnq,dddbq	C	C	C	C	P	P	P	P	P	P	C	C
43	dddddnq,ddddbq	C	C	C	C	P	P	P	P	P	P	C	C
44	ddddnq,b(q,g)	P	C	C	P	P	P	P	P	P	P	C	P
45	ddddnq,bbp	P	C	C	P	P	P	P	P	P	P	C	P
46	ddddnq,bq	P	C	C	P	P	P	P	P	P	P	C	P
47	ddddnq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
48	ddddnq,dddbq	C	C	C	C	P	P	P	P	P	P	C	C
49	ddddnq,dp,bbp	P	C	C	P	P	P	P	P	P	P	C	P
50	dddnq,bbq	P	C	C	P	P	P	P	P	P	P	C	P
51	dddnq,bq	P	C	C	P	P	P	P	P	P	P	C	P

52	dddnq,dbq	C	C	C	C	P	P	P	P	P	P	C	C
53	dddnq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
54	dddnq,dddbq	C	C	C	C	P	P	P	P	P	P	C	C
55	dddnq,ddddbq	C	C	C	C	P	P	P	P	P	P	C	C
56	dddnq,dp,bbp	P	C	C	P	P	P	P	P	P	P	C	P
57	ddnq,bq	P	C	C	P	P	P	P	P	P	P	C	P
58	ddnq,dbq	C	C	C	C	P	P	P	P	P	P	C	C
59	ddnq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
60	ddnq,dddbq	C	C	C	C	P	P	P	P	P	P	C	C
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P
62	Nnd	C	C	C	C	C	C	C	C	C	C	C	C
63	dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P
64	dnq,dbq	C	C	C	C	P	P	P	P	P	P	C	C
65	dnq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
66	dnq,dddbq	C	C	C	C	P	P	P	P	P	P	C	C
67	dnq,ddp,bbbq	P	P	P	C	P	P	P	P	P	P	P	C
68	n(bp,dp)	P	P	P	P	C	C	P	P	P	P	P	P
69	n(d1d2b3bp->p)	C	P	P	C	P	P	P	P	P	P	P	C
70	n(dbbbbp->bbdp)	C	P	P	C	P	P	P	P	P	P	P	C
71	n(dbbp->bp)	C	P	P	C	P	P	P	P	P	P	P	C
72	n(dbdnp->nbbp)	P	P	C	P	P	P	P	P	P	P	C	P
73	n(dbp->dp)	P	P	C	P	P	P	P	P	P	P	C	P
74	n(ddbbp->p)	C	P	P	C	P	P	P	P	P	P	P	C
75	n(dp->p)	C	C	C	C	C	C	C	C	C	C	C	C
76	nd,g	P	P	P	P	C	C	P	P	P	P	P	P
77	nd	P	P	P	P	C	C	P	P	P	P	P	P
78	np,bbbp,dddddddq	P	P	C	C	P	P	P	P	P	P	C	C
79	np,bbp,dddddddq	P	P	P	C	P	P	P	P	P	P	P	C
80	np,ddddddddbp	C	C	C	C	P	P	P	P	P	P	C	C
81	np,dddddddddbp	C	C	C	C	C	C	C	C	C	C	C	C
82	nq,ddbq	C	C	C	C	P	P	P	P	P	P	C	C
83	nq,dddddbq	C	C	C	C	P	P	P	P	P	P	C	C
84	p,bbndp,ddq	P	P	C	C	P	P	P	P	P	P	C	C
85	(db)^1p,p	C	P	P	C	P	P	P	P	P	P	P	C
86	(db)^1p,q	C	C	C	C	C	C	C	C	C	C	C	C
87	(db)^2p,p	C	P	P	C	P	P	P	P	P	P	P	C
88	(db)^2p,q	C	C	C	C	C	C	C	C	C	C	C	C
89	(db)^3p,p	C	P	P	C	P	P	P	P	P	P	P	C
90	(db)^3p,q	C	C	C	C	C	C	C	C	C	C	C	C
91	(db)^4p,p	C	P	P	C	P	P	P	P	P	P	P	C
92	(db)^4p,q	C	C	C	C	C	C	C	C	C	C	C	C
93	(db)^5p,p	C	P	P	C	P	P	P	P	P	P	P	C
94	(db)^5p,q	C	C	C	C	C	C	C	C	C	C	C	C
95	(db)^6p,p	C	P	P	C	P	P	P	P	P	P	P	C
96	(db)^6p,q	C	C	C	C	C	C	C	C	C	C	C	C
97	(db)^7p,p	C	P	P	C	P	P	P	P	P	P	P	C
98	(db)^7p,q	C	C	C	C	C	C	C	C	C	C	C	C
99	(db)^8p,p	C	P	P	C	P	P	P	P	P	P	P	C
100	(db)^8p,q	C	C	C	C	C	C	C	C	C	C	C	C
101	(db)^9p,p	C	P	P	C	P	P	P	P	P	P	P	C
102	(db)^9p,q	C	C	C	C	C	C	C	C	C	C	C	C
103	(db)^10p,p	C	P	P	C	P	P	P	P	P	P	P	C
104	(db)^10p,q	C	C	C	C	C	C	C	C	C	C	C	C
105	aiml02_prop3i	P	C	C	C	P	P	P	P	P	P	C	C
106	aiml02_prop3ii	P	C	C	P	P	P	P	P	P	P	C	P
107	aiml02_prop3iii	C	C	C	C	P	P	P	P	P	P	C	C
108	amai02	P	C	C	P	P	P	P	P	P	P	C	P
109	amai02b	C	C	C	C	P	P	P	P	P	P	C	C
110	demri1	P	C	C	C	P	P	P	P	P	P	C	C
111	demri2	P	C	C	P	P	P	P	P	P	P	C	P
112	demri3	P	P	P	P	P	P	P	P	P	P	P	P
113	demri5	P	C	C	P	C	C	P	P	P	P	C	P
114	demri6	C	C	C	C	C	C	C	C	C	C	C	C
115	demri7	P	C	C	C	P	P	P	P	P	P	C	C
116	demri8	P	C	C	C	P	P	P	P	P	P	C	C
117	demri9	C	C	C	C	C	C	C	C	C	C	C	C
118	CR*	C	C	C	C	C	C	C	C	C	C	C	C
119	demri4*	P	P	P	P	P	X	P	P	P	P	P	P
120	bbf	P	P	P	P	C	C	P	P	P	P	P	P
121	bbt	C	C	C	C	C	C	C	C	C	C	C	C
122	bdf	P	P	P	P	C	C	P	P	P	P	P	P
123	bdt	C	C	C	C	C	C	C	C	C	C	C	C
124	bf	P	P	P	P	C	C	P	P	P	P	P	P
125	bt	C	C	C	C	C	C	C	C	C	C	C	C
126	dbf	P	P	P	P	P	P	P	P	P	P	P	P
127	dbt	C	C	C	C	C	C	C	C	C	C	C	C
128	ddf	P	P	P	P	P	P	P	P	P	P	P	P
129	ddt	C	C	C	C	C	C	C	C	C	C	C	C
130	df	P	P	P	P	P	P	P	P	P	P	P	P

131	dt	C	C	C	C	C	C	C	C	C	C	C	C
132	fa	P	P	P	P	P	P	P	P	P	P	P	P
133	tr	C	C	C	C	C	C	C	C	C	C	C	C

*Axiom applied to all the modality indices in the input problem.

Figure 7.7 **<u>Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed single axioms</u>.** The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. Again, this data is a composite of two sets of experiments (see figure 7.5 for more details). The total execution time for the data in this table is 67.5 hours (2*67.5 hours).

Software testing: It was confirmed that, where a result (Proof or Completion) was produced for both the translation of a particular formula with the axiom translated as correspondence property and axiom translated as an axiomatic schema, that there was *no difference* between these outcomes in any of the cases tested (the outcomes for T = the outcomes for T_c; $5_o = 5_c$, etc).

No.	Target Formula	T_c	D_c	B_c	4_c	5_c	alt_{1c}	$4^2{}_c$	$4^3{}_c$	$5^2{}_c$	$5^3{}_c$	$alt_1{}^{1,1}{}_c$	$alt_1{}^{2,1}{}_c$	$alt_1{}^{1,2}{}_c$	$alt_1{}^{2,2}{}_c$
1	4	C	C	C	P	X	X	X	X	X	X	X	X	X	X
2	4^2	C	C	C	P	X	X	P	X	X	X	X	X	X	X
3	4^3	C	C	C	P	X	X	X	P	X	X	X	X	X	X
4	5	C	C	C	X	P	X	X	X	X	X	X	X	X	X
5	5^2	C	C	C	X	P	X	X	X	P	X	X	X	X	X
6	5^3	C	C	C	X	P	X	X	X	X	P	X	X	X	X
7	D	P	P	C	X	X	X	X	X	X	X	X	X	X	X
8	T	P	C	C	X	X	X	X	X	X	X	X	X	X	X
9	alt1	C	C	C	X	X	P	X	X	X	X	X	X	X	X
10	alt1^1,1	C	C	C	X	X	P	X	X	X	X	P	X	X	X
11	alt1^1,2	C	C	C	X	X	X	X	X	X	X	X	X	X	X
12	alt1^2,1	C	C	C	X	X	X	X	X	X	X	X	X	X	X
13	alt1^2,2	C	C	C	X	X	P	X	X	X	X	P	X	X	X
14	B	C	C	P	X	X	X	X	X	X	X	X	X	X	X
15	B^2	C	C	C	X	X	X	X	X	X	X	X	X	X	X
16	F	C	C	C	X	X	X	X	X	X	X	X	X	X	X
17	M	C	C	C	X	X	X	X	X	X	X	X	X	X	X
18	Cxt	C	C	C	X	P	P	X	X	P	X	X	X	X	X
19	bbp,dddd(q,dnp)	C	C	C	P	X	X	X	X	X	X	X	X	X	X
20	bp,dddnp,g	C	C	C	P	X	X	P	X	X	X	X	X	X	X
21	bp,dddp,g	C	C	C	X	X	X	X	X	X	X	X	X	X	X
22	bp,ddnp,g	C	C	C	P	X	X	X	X	X	X	X	X	X	X
23	bp,ddp	C	C	C	X	X	X	X	X	X	X	X	X	X	X
24	bq,d	C	C	C	X	X	X	X	X	X	X	X	X	X	X
25	bq,ddddddnq	C	C	C	P	X	X	P	X	X	X	X	X	X	X
26	bq,ddddnq	C	C	C	P	X	X	P	X	X	X	X	X	X	X
27	d(bp,ddnq),ddbq	C	C	C	X	P	P	X	X	X	X	P	X	X	X
28	d(dddddnq,ddddbq)	C	C	C	X	P	P	X	X	X	X	X	X	X	X
29	d(ddddnq,dddbq)	C	C	C	X	P	P	X	X	X	X	X	X	X	X
30	d(ddnq,dbq)	C	C	C	X	P	P	X	X	P	X	P	X	X	X
31	d(dnq,dbq)	C	C	C	X	P	X	X	X	X	X	X	X	X	X
32	d(dnq,dddbq)	C	C	C	X	P	X	X	X	X	X	X	X	X	X
33	dbp,dd(np,bq)	C	C	C	X	P	P	X	X	P	X	X	X	X	X
34	dbp,dddddnp	C	C	C	X	P	X	X	X	X	X	X	X	X	X
35	dbp,ddnp	C	C	C	X	P	P	X	X	P	X	X	X	X	X
36	dbq,d(dnq,ddp)	C	C	C	X	P	P	X	X	P	X	X	X	X	X
37	dd(bq,ddddnq)	C	C	C	P	P	X	X	X	X	X	X	X	X	X
38	dd(dnq,dbq)	C	C	C	X	P	X	X	X	X	X	X	P	P	X
39	ddddddnq,dp,bbp	C	C	C	P	X	X	P	X	X	X	X	X	X	X
40	dddddnq,bbbq	C	C	C	P	P	X	P	X	X	X	X	X	X	X
41	dddddnq,ddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
42	dddddnq,dddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
43	dddddnq,ddddbq	C	C	C	X	P	P	X	X	X	X	X	X	X	X
44	ddddnq,b(q,g)	C	C	C	P	X	X	X	P	X	X	X	X	X	X
45	ddddnq,bbp	C	C	C	P	P	X	P	X	X	X	X	P	X	X
46	ddddnq,bq	C	C	C	P	X	X	X	P	X	X	X	X	X	X
47	ddddnq,ddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X

48	ddddnq,dddbq	C	C	C	X	P	P	X	X	X	X	P	X	X	X
49	ddddnq,dp,bbp	C	C	C	P	X	X	P	X	X	X	X	X	X	X
50	dddnq,bbq	C	C	C	P	P	X	X	X	X	X	X	P	X	X
51	dddnq,bq	C	C	C	P	X	X	P	X	X	X	X	X	X	X
52	dddnq,dbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
53	dddnq,ddbq	C	C	C	X	P	P	X	X	P	X	P	X	X	X
54	dddnq,dddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
55	dddnq,ddddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
56	dddnq,dp,bbp	C	C	C	P	X	X	X	X	X	X	X	P	X	X
57	ddnq,bq	C	C	C	P	X	X	X	X	X	X	X	X	X	X
58	ddnq,dbq	C	C	C	X	P	P	X	X	P	X	X	X	X	X
59	ddnq,ddbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
60	ddnq,dddbq	C	C	C	X	P	X	X	X	P	X	X	X	X	X
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
62	Nnd	C	C	C	X	X	X	X	X	X	X	X	X	X	X
63	dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
64	dnq,dbq	C	C	C	X	P	X	X	X	X	X	X	X	X	X
65	dnq,ddbq	C	C	C	X	P	X	X	X	P	X	X	X	X	X
66	dnq,dddbq	C	C	C	X	P	X	X	X	X	P	X	X	X	X
67	dnq,ddp,bbbq	P	C	P	X	P	X	X	X	P	X	X	X	X	X
68	n(bp,dp)	P	P	C	X	X	X	X	X	X	X	X	X	X	X
69	n(d1d2b3bp->p)	C	C	P	X	X	X	X	X	X	X	X	X	X	X
70	n(dbbbbp->bbdp)	C	C	P	X	P	X	X	X	P	X	X	X	X	X
71	n(dbbp->bp)	C	C	P	X	P	X	X	X	X	X	X	X	X	X
72	n(dbdnp->nbbp)	P	C	C	X	P	X	X	X	X	X	X	X	X	X
73	n(dbp->dp)	P	C	C	X	P	X	X	X	X	X	X	X	X	X
74	n(ddbbp->p)	C	C	P	X	X	X	X	X	X	X	X	X	X	X
75	n(dp->p)	C	C	C	X	X	X	X	X	X	X	X	X	X	X
76	nd,g	P	P	C	X	X	X	X	X	X	X	X	X	X	X
77	nd	P	P	C	X	X	X	X	X	X	X	X	X	X	X
78	np,bbbp,ddddddddq	P	C	C	X	X	X	X	X	X	X	X	X	X	X
79	np,bbp,ddddddddq	P	C	P	X	X	X	X	X	X	X	X	X	X	X
80	np,ddddddddbp	C	C	C	X	X	X	X	X	X	X	X	X	X	X
81	np,dddddddddbp	C	C	C	X	X	X	X	X	X	X	X	X	X	X
82	nq,ddbq	C	C	C	X	X	X	X	X	X	X	X	X	X	X
83	nq,dddddbq	C	C	C	X	X	X	X	X	X	X	X	X	X	X
84	p,bbndp,ddq	P	C	C	X	X	X	X	X	X	X	X	X	X	X
85	(db)^1p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
86	(db)^1p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
87	(db)^2p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
88	(db)^2p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
89	(db)^3p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
90	(db)^3p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
91	(db)^4p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
92	(db)^4p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
93	(db)^5p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
94	(db)^5p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
95	(db)^6p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
96	(db)^6p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
97	(db)^7p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
98	(db)^7p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
99	(db)^8p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
100	(db)^8p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
101	(db)^9p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
102	(db)^9p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
103	(db)^10p,p	C	C	P	X	X	X	X	X	X	X	X	X	X	X
104	(db)^10p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
105	aiml02_prop3i	C	C	C	X	P	X	X	X	X	X	X	X	X	X
106	aiml02_prop3ii	C	C	C	P	P	X	X	X	X	X	X	X	X	X
107	aiml02_prop3iii	C	C	C	X	P	X	X	X	X	X	X	X	X	X
108	amai02	C	C	C	P	X	X	X	X	X	X	X	X	X	X
109	amai02b	C	C	C	X	P	X	X	X	X	X	X	X	X	X
110	demri1	C	C	C	X	X	X	X	X	X	X	X	X	X	X
111	demri2	C	C	C	P	X	X	X	X	X	X	X	X	X	X
112	demri3	P	P	P	P	P	P	P	P	P	P	P	P	P	P
113	demri5	C	C	C	X	X	X	X	X	X	X	X	X	X	X
114	demri6	C	C	C	X	X	X	X	X	X	X	X	X	X	X
115	demri7	C	C	C	X	P	X	X	X	X	X	X	X	X	X
116	demri8	C	C	C	X	P	X	X	X	X	X	X	X	X	X
117	demri9	C	C	C	X	X	X	X	X	X	X	X	X	X	X
118	CR*	C	C	C	X	X	X	X	X	X	X	X	X	X	X
119	demri4*	P	P	P	X	X	X	X	X	X	X	X	X	X	X
120	bbf	P	P	C	X	X	C	X	X	X	X	C	C	X	C
121	bbt	C	C	C	C	C	C	C	C	C	C	C	C	C	C
122	bdf	P	P	C	X	X	C	X	X	X	C	C	C	C	C
123	bdt	C	C	C	X	X	C	X	X	X	X	C	C	C	C
124	bf	P	P	C	X	X	C	X	X	X	C	C	C	C	C
125	bt	C	C	C	C	C	C	C	C	C	C	C	C	C	C
126	dbf	P	P	P	X	P	C	X	X	X	X	C	C	C	X

127	dbt	C	C	C	X	X	C	X	X	X	X	C	C	C	C
128	ddf	P	P	P	P	P	P	P	P	P	P	P	P	P	P
129	ddt	C	C	C	X	X	C	X	X	X	X	C	C	C	X
130	df	P	P	P	P	P	P	P	P	P	P	P	P	P	P
131	dt	C	C	C	X	X	C	X	X	X	X	C	C	C	C
132	fa	P	P	P	P	P	P	P	P	P	P	P	P	P	P
133	tr	C	C	C	C	C	C	C	C	C	C	C	C	C	C

*Axiom applied to all the modality indices in the input problem.

Figure 7.8 <u>Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combination</u>s. The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. Again, this data is a composite of two sets of experiments (see figure 7.5 for more details). The total execution time for the data in his table is 24.6 hours (2*24.6 hours).

Software testing: Again, it was confirmed that, where a result (Proof or Completion) was produced for both the translation of a particular formula with the axiom translated as correspondence property and axiom translated as an axiomatic schema, that there was *no difference* between these outcomes in any of the cases tested (the outcomes for T4 = the outcomes for T_c4_c, etc).

No.	Target Formula	T_c4_c	T_cB_c	D_cB_c	D_c4_c	4_cB_c	5_cB_c	T_c5_c	$T_c4_cB_c$	$D_c4_cB_c$
1	4	P	C	C	P	P	P	P	P	X
2	4^2	P	C	C	P	X	X	X	X	X
3	4^3	P	C	C	P	X	X	X	X	X
4	5	X	C	C	X	P	P	P	P	P
5	5^2	X	C	C	X	P	P	P	P	P
6	5^3	X	C	C	X	P	P	P	P	P
7	D	P	P	P	P	X	X	P	P	P
8	T	P	P	C	X	X	X	P	P	P
9	alt1	X	C	C	X	X	X	X	X	X
10	alt1^1,1	X	C	C	X	X	X	X	X	X
11	alt1^1,2	X	C	C	X	X	X	X	X	X
12	alt1^2,1	X	C	C	X	X	X	X	X	X
13	alt1^2,2	X	C	C	X	X	X	X	X	X
14	B	X	P	P	X	P	P	P	P	P
15	B^2	X	C	C	X	P	P	P	P	P
16	F	X	C	C	X	X	X	X	X	X
17	M	X	C	C	X	X	X	X	X	X
18	Cxt	X	C	C	X	P	P	P	P	P
19	bbp,dddd(q,dnp)	P	C	C	P	X	X	X	X	X
20	bp,dddnp,g	X	C	C	P	X	X	X	X	X
21	bp,dddp,g	X	C	C	X	X	X	X	X	X
22	bp,ddnp,g	X	C	C	X	X	X	X	X	X
23	bp,ddp	X	C	C	X	X	X	X	X	X
24	bq,d	X	C	C	X	X	X	X	X	X
25	bq,ddddddddnq	X	C	C	P	X	X	X	X	X
26	bq,dddddnq	P	C	C	P	X	X	X	X	X
27	d(bp,ddnq),ddbq	X	C	C	X	P	P	P	P	X
28	d(dddddnq,ddddbq)	X	C	C	X	P	P	P	X	X
29	d(ddddnq,dddbq)	X	C	C	X	P	P	P	P	X
30	d(ddnq,dbq)	X	C	C	X	P	P	P	P	P
31	d(dnq,dbq)	X	C	C	X	P	P	P	P	P
32	d(dnq,dddbq)	X	C	C	X	P	P	P	P	P
33	dbp,dd(np,bq)	X	C	C	X	P	P	P	P	P
34	dbp,dddddnp	X	C	C	X	P	P	P	P	P
35	dbp,ddnp	X	C	C	X	P	P	P	P	P
36	dbq,d(dnq,ddp)	X	C	C	X	P	P	P	P	P
37	dd(bq,ddddnq)	P	C	C	P	P	P	P	X	X
38	dd(dnq,dbq)	X	C	C	X	P	P	P	P	P
39	ddddddnq,dp,bbp	X	C	C	P	X	X	X	X	X
40	dddddnq,bbbq	P	C	C	P	X	X	P	X	X
41	dddddnq,ddbq	X	C	C	X	P	P	P	P	P
42	dddddnq,dddbq	X	C	C	X	P	P	P	X	X
43	dddddnq,ddddbq	X	C	C	X	P	P	P	X	X

44	ddddnq,b(q,g)	P	C	C	P	X	X	X	X	X
45	ddddnq,bbp	P	C	C	P	X	X	X	X	X
46	ddddnq,bq	P	C	C	P	X	X	X	X	X
47	ddddnq,ddbq	X	C	C	X	P	P	P	P	P
48	ddddnq,dddbq	X	C	C	X	P	P	P	P	X
49	ddddnq,dp,bbp	P	C	C	P	X	X	X	X	X
50	dddnq,bbq	P	C	C	P	X	P	P	X	X
51	dddnq,bq	P	C	C	P	X	X	X	X	X
52	dddnq,dbq	X	C	C	X	P	P	P	P	X
53	dddnq,ddbq	X	C	C	X	P	P	P	P	P
54	dddnq,dddbq	X	C	C	X	P	P	P	P	X
55	dddnq,ddddbq	X	C	C	X	P	P	P	X	X
56	dddnq,dp,bbp	P	C	C	P	X	P	P	X	X
57	ddnq,bq	P	C	C	P	P	P	P	P	X
58	ddnq,dbq	X	C	C	X	P	P	P	P	P
59	ddnq,ddbq	X	C	C	X	P	P	P	P	P
60	ddnq,dddbq	X	C	C	X	P	P	P	P	X
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P
62	Nnd	X	C	C	X	X	X	X	X	X
63	dnq,bq	P	P	P	P	P	P	P	P	P
64	dnq,dbq	X	C	C	X	P	P	P	P	P
65	dnq,ddbq	X	C	C	X	P	P	P	P	P
66	dnq,dddbq	X	C	C	X	P	P	P	P	P
67	dnq,ddp,bbbq	P	P	P	X	P	P	P	P	P
68	n(bp,dp)	P	P	P	P	X	X	P	P	P
69	n(d1d2b3bp->p)	X	P	P	X	P	P	P	P	P
70	n(dbbbbp->bbdp)	X	P	P	X	P	P	P	P	P
71	n(dbbp->bp)	X	P	P	X	P	P	P	P	P
72	n(dbdnp->nbbp)	P	P	C	P	P	P	P	P	P
73	n(dbp->dp)	P	P	C	P	P	P	P	P	P
74	n(ddbbp->p)	X	P	P	X	P	P	P	P	P
75	n(dp->p)	X	C	C	X	X	X	X	X	X
76	nd,g	P	P	P	P	X	X	P	P	P
77	nd	P	P	P	P	X	X	P	P	P
78	np,bbbp,dddddddq	P	P	C	X	P	P	P	P	P
79	np,bbp,ddddddddq	P	P	P	X	P	P	P	P	P
80	np,ddddddddbp	X	C	C	X	P	P	P	P	P
81	np,dddddddddbp	X	C	C	X	X	X	X	X	X
82	nq,ddbq	X	C	C	X	P	P	P	P	P
83	nq,dddddbq	X	C	C	X	P	P	P	P	P
84	p,bbndp,ddq	P	P	C	X	P	P	P	P	P
85	(db)^1p,p	X	P	P	X	P	P	P	P	P
86	(db)^1p,q	X	C	C	X	X	X	X	X	X
87	(db)^2p,p	X	P	P	X	P	P	P	P	P
88	(db)^2p,q	X	C	C	X	X	X	X	X	X
89	(db)^3p,p	X	P	P	X	P	P	P	P	P
90	(db)^3p,q	X	C	C	X	X	X	X	X	X
91	(db)^4p,p	X	P	P	C	P	P	P	P	P
92	(db)^4p,q	X	C	C	C	X	X	X	C	X
93	(db)^5p,p	X	P	P	C	P	P	P	P	P
94	(db)^5p,q	X	C	C	C	X	X	X	C	X
95	(db)^6p,p	X	P	P	C	P	P	P	P	P
96	(db)^6p,q	X	C	C	C	X	X	X	C	X
97	(db)^7p,p	X	P	P	C	P	P	P	P	P
98	(db)^7p,q	X	C	C	C	X	X	X	C	X
99	(db)^8p,p	X	P	P	C	P	P	P	P	P
100	(db)^8p,q	X	C	C	C	X	X	X	C	X
101	(db)^9p,p	X	P	P	C	P	P	P	P	P
102	(db)^9p,q	X	C	C	C	X	X	X	C	X
103	(db)^10p,p	X	P	P	C	P	P	P	P	P
104	(db)^10p,q	X	C	C	C	X	X	X	C	X
105	aiml02_prop3i	P	C	C	X	P	P	P	P	P
106	aiml02_prop3ii	X	C	C	X	P	P	P	P	X
107	aiml02_prop3iii	X	C	C	X	P	P	P	P	P
108	amai02	P	C	C	P	P	P	P	P	X
109	amai02b	X	C	C	X	P	P	P	P	P
110	demri1	P	C	C	X	X	X	X	X	X
111	demri2	P	C	C	X	P	P	P	X	X
112	demri3	P	P	P	P	P	P	P	P	X
113	demri5	P	C	C	X	X	X	X	X	X
114	demri6	X	C	C	X	X	X	X	X	X
115	demri7	P	C	C	X	P	P	P	P	P
116	demri8	X	C	C	X	P	P	P	P	P
117	demri9	X	C	C	X	X	X	X	X	X
118	CR*	X	C	C	X	X	X	X	X	X
119	demri4*	X	P	P	X	X	X	X	X	X
120	bbf	P	P	P	P	C	C	P	P	P
121	bbt	C	C	C	C	C	C	C	C	C
122	bdf	P	P	P	P	X	X	P	P	P

123	bdt	X	C	C	C	X	X	X	C	X
124	bf	P	P	P	P	X	X	P	P	P
125	bt	C	C	C	C	C	C	C	C	C
126	dbf	P	P	P	P	P	P	P	P	P
127	dbt	X	C	C	C	X	X	X	C	X
128	ddf	P	P	P	P	P	P	P	P	P
129	ddt	X	C	C	C	X	X	X	C	X
130	df	P	P	P	P	P	P	P	P	P
131	dt	X	C	C	C	X	X	X	C	X
132	fa	P	P	P	P	P	P	P	P	P
133	tr	C	C	C	C	C	C	C	C	C

*Axiom applied to all the modality indices in the target formula.

Figure 7.9 **Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations: Execution times.** The data is taken from the results for test problems 1-119 (see figure 7.4 for details), and is an analysis of the same experiments reported in figures 7.5 to 7.8 (except that only one set of data was analyzed; see figure 7.5 for more details). Problems are deemed to have failed if they were not solved before running out of time at 200 seconds execution time. Execution time statistics are listed for the solved problems. The statistics for all the problems, regardless of outcome, are all listed under the heading *Total*. This set of solved problems is sub-divided by the outcome, either Completion Found in SPASS (satisfiable), or Proof Found in SPASS (unsatisfiable), in the other two sections. Where there is no data (no problems solved) the table is blank. The median, arithmetic mean ± standard deviation, and maximum values are recorded.

	Total set of problems				Problem outcome = Proof				Problem outcome = Completion			
Axiom	No. problems Solved (failed)	Execution Time (msec) Median	Mean±SD	Max	No. problems solved	Execution Time (msec) Median	Mean±SD	Max	No. problems solved	Execution Time (msec) Median	Mean±SD	Max
K	119(0)	20	27±27	160	4	15	28±29	70	115	20	27±27	160
T	119(0)	40	57±34	200	15	40	39±15	80	104	50	59±35	200
B	119(0)	20	91±318	2960	21	20	52±100	480	98	20	99±347	2960
D	119(0)	30	97±178	1050	8	10	54±112	330	111	30	100±182	1050
4	119(0)	20	288±1140	8230	26	20	24±18	90	93	20	361±1281	8230
5	119(0)	60	510±1802	16250	53	50	91±121	630	66	110	847±2372	16250
alt_1	119(0)	30	117±222	1290	19	30	48±87	400	100	40	131±237	1290
4^2	119(0)	40	5194±24785	185280	13	20	44±69	270	106	45	5825±26204	185280
4^3	94(25)	155	11972±33607	158360	10	35	772±2325	7390	84	310	13306±35327	158360
5^2	74(45)	555	10568±27215	193690	27	180	1022±3449	18120	47	2910	16051±32931	193690
5^3	23(96)	1140	7494±21355	102720	17	1770	9373±24644	102720	6	590	2170±4147	10560
$alt_1^{1,1}$	90(29)	180	11428±27917	139520	12	45	62±53	180	78	670	13177±29624	139520
$alt_1^{2,1}$	85(34)	280	8563±21385	117540	30	115	2665±12537	68900	55	560	11781±24436	117540
$alt_1^{1,2}$	67(52)	400	6194±20623	142010	24	380	5158±17993	85240	43	400	6773±22138	142010
$alt_1^{2,2}$	27(92)	4050	8348±13845	52120	8	1830	10204±18318	52120	19	6390	7567±12009	49740
T4	119(0)	20	70±170	1110	42	20	26±19	100	77	20	94±208	1110
TB	119(0)	20	67±206	1910	30	20	29±26	100	89	20	79±236	1910
DB	119(0)	40	877±5255	52470	25	40	103±150	550	94	40	1084±5902	52470
D4	119(0)	40	454±1663	11790	33	30	51±78	420	86	50	609±1936	11790
4_oB	119(0)	40	379±1449	11190	87	30	143±823	7710	32	60	1021±2352	11190
5_oB	118(1)	50	1229±9201	97430	86	50	93±137	930	32	95	4282±17501	97430

5_oT	119(0)	50	310±1495	14360	93	40	149±759	7350	26	95	885±2827	14360
$T4_oB$	119(0)	40	408±1832	16570	93	30	71±185	1710	26	85	1614±3711	16570
D_o4_oB	116(3)	130	3729±16585	116210	93	110	1474±12032	116210	23	270	12845±26907	109570
T_c4B_c	119(0)	30	156±490	4260	93	20	58±105	730	26	55	507±963	4260
D_cB	119(0)	30	102±330	3080	25	20	35±40	200	94	30	119±370	3080
D_c4	119(0)	30	103±246	1670	33	20	32±36	170	86	30	131±285	1670

	Total set of problems				**Problem outcome = Proof**				**Problem outcome = Completion**			
Axiom	**No. Problems Solved (failed)**	**Execution Time (msec) Median**	**Mean±SD**	**Max**	**No. Problems Solved**	**Execution Time (msec) Median**	**Mean±SD**	**Max**	**No. Problems Solved**	**Execution Time (msec) Median**	**Mean±SD**	**Max**
T_c	119(0)	20	30±28	200	15	10	28±48	200	104	20	31±24	120
B_c	119(0)	30	107±304	2720	21	30	46±43	160	98	30	121±334	2720
D_c	119(0)	20	28±23	120	8	10	24±35	110	111	20	28±23	120
4_c	25(94)	70	464±1221	6110	25	70	464±1221	6110	0	–	–	–
5_c	48(71)	115	424±820	3690	48	115	424±820	3690	0	–	–	–
Alt_{1c}	18(101)	40	55±45	180	18	40	55±45	180	0	–	–	–
4^2_c	12(107)	595	19965±40528	120080	12	595	19965±40528	120080	0	–	–	–
4^3_c	6(113)	16700	23457±29637	73950	6	16700	23457±29637	73950	0	–	–	–
5^2_c	15(104)	1140	23082±36088	101480	15	1140	23082±36088	101480	0	–	–	–
5^3_c	5(114)	90	78222±107069	197470	5	90	78222±107069	197470	0	–	–	–
$Alt_1^{1,1}{}_c$	9(110)	8980	31962±52804	159310	9	8980	31962±52804	159310	0	–	–	–
$Alt_1^{2,1}{}_c$	7(112)	1050	46879±79159	169940	7	1050	46879±79159	169940	0	–	–	–
$Alt_1^{1,2}{}_c$	4(115)	70	29043±57985	116020	4	70	29043±57985	116020	0	–	–	–
$Alt_1^{2,2}{}_c$	3(116)	10	57±81	150	3	10	57±81	150	0	–	–	–
T_c4_c	36(83)	110	10392±33343	180490	36	110	10392±33343	180490	0	–	–	–
T_cB_c	119(0)	30	83±174	1200	30	20	57±122	670	89	30	91±188	1200
D_cB_c	119(0)	30	100±319	2920	25	20	35±38	190	94	30	118±357	2920
D_c4_c	42(77)	665	5545±14455	66820	28	1135	8042±17256	66820	14	360	551±517	1560
4_cB_c	69(50)	120	466±1062	6160	69	120	466±1062	6160	0	–	–	–
5_cB_c	71(48)	80	2363±12054	89040	71	80	2363±12054	89040	0	–	–	–
T_c5_c	77(42)	140	481±1677	14280	77	140	481±1677	14280	0	–	–	–
$T_c4_cB_c$	68(51)	90	1850±12338	101890	68	90	1850±12338	101890	0	–	–	–
$D_c4_cB_c$	57(62)	320	3322±11751	80490	57	320	3322±11751	80490	0	–	–	–

Figure 7.10 Axiomatic translation of the test formulae: Summary of counter-examples: The formulae that show discrimination between potentially complete and non-complete formulations of the axiom combination are shown (that is, the complete formulation yields the result Proof or Completion, and the incomplete formulation yields the result Completion or Proof, respectively). Formulations of axiom combinations showing non-complete examples are in brackets. Refer to figure 7.4 for the actual formula. Formulae 1-117 were screened. The total execution time for the data in this table is 68.2 hours.

Discrimination between formulation of modal axioms	**Discriminating Formulae (counter-examples)**
[K5] vs $K5_o$	bbp,dddd(q,dnp); ddddddnq,dp,bbp; dddddnq,bbbq; ddddnq,bbp; ddddnq,dp,bbp; dddnq,bbq; dddnq,dp,bbp
[5^2] vs 5^2_o	dddddddnq,dp,bbp; ddddddnq,bbbq; dddddnq,bbp; ddddnq,dp,bbp

[5^3] vs 5^3_o	None
[4B / $B_o 4$] vs $4_o B$	5; aiml02_prop3iii; amai02b; dnq,dbq
$4_o B_o 5$ / $4_o 5_o B$ / $5_o B_o 4$ / $5_o 4_o B$ / $B_o 4_o 5$ / $B_o 5_o 4$	None
$B_o T$ / $T_o B$ / BT	None
D45 / $D_o 4_o 5_o$ / $D_o 5_o 4$ / $4_o D_o 5_o$ / $4_o 5_o D$ / $5_o D_o 4$ / $5_o 4_o D$	None
[$B_o 5_o$] vs $5_o B$	4; 4^2; 4^3; amai02; bp,dddnp,g; bq,dddddddnq; bq,dddddnq; ddddnq,b(q,g); ddddnq,bq; dddnq,bq; ddnq,bq; demri1; demri2
[T4B / $TB_o 4$ / $B_o 4_o T$ / $B_o T4$] vs $T4_o B$ / $4_o B_o T$ / $4_o TB$	5; aiml02_prop3iii; amai02b; dnq,dbq
DT / $D_o T$ / TD	None
[$T5_o$] vs $5_o T$	4; 4^2; 4^3; amai02; bp,dddnp,g; bq,dddddddnq; bq,dddddnq; ddddnq,b(q,g); ddddnq,bq; dddnq,bq; ddnq,bq; demri1; demri2
[$TB_o 5_o$] vs $B_o 5_o T$ / $T5_o B$ / $5_o B_o T$ / $5_o TB$ / $B_o T5_o$	4; 4^2; 4^3; amai02; bp,dddnp,g; bq,dddddddnq; bq,dddddnq; ddddnq,b(q,g); ddddnq,bq; dddnq,bq; ddnq,bq; demri1; demri2
[$B_o 5_o D$ / $B_o D_o 5_o$ / $D_o B_o 5_o$ *]* vs $D_o 5_o B$ / $5_o D_o B$ / $5_o B_o D$	4; 4^2; 4^3; amai02; bq,dddddddnq; bq,dddddnq; ddddnq,b(q,g); ddddnq,bq; dddnq,bq; ddnq,bq; demri1 bp,dddnp,g; dddnq,bbq; demri2 ; T
[D4B / $B_o D_o 4$ / $B_o 4_o D$] vs $D_o B_o 4$/ $D_o 4_o B$ / $4_o D_o B$ / $4_o B_o D$	5; aiml02_prop3iii; amai02b; dnq,dbq

Figure 7.11 **<u>Summary of the results for axiomatic translation of formulae in local satisfiability calculations in various formulations of axiom S5</u>**. The test data consists of formulae 1-117. The S5 formulations examined are listed below. Formulations for which counter-examples were identified (and hence are not complete) are marked with a strike-through notation.

KT4B: $T_c 4_c B_c$ ~~T4B~~ $T_o 4_o B$ ~~$T_o B_o 4$~~ ~~$B_o 4_o T$~~ ~~$B_o T_o 4$~~ $4_o T_o B$ $4_o B_o T$
KD4B: $D_c 4_c B_c$ ~~D4B~~ $D_o 4_o B$ $D_o B_o 4$ ~~$B_o D_o 4$~~ ~~$B_o 4_o D$~~ $4_o D_o B$ $4_o B_o D$
KT5: ~~$T5_o$~~ $T_c 5_c$ $5_o T$
KTB5: $T_c B_c 5_c$ ~~$T_o B_o 5_o$~~ $T_o 5_o B$ $5_o B_o T$ $5_o T_o B$ $B_o T_o 5_o$ $B_o 5_o T$
KT4B5: $T_c 4_c B_c 5_c$ $4_o TB_o 5_o$ $4_o T5_o B$ $4_o 5_o B_o T$ $4_o 5_o TB$ $4_o B_o T5_o$ $4_o B_o 5_o T$
$B_o 4_o T5_o$ $B_o 4_o 5_o T$ $B_o 5_o T4$ $B_o 5_o 4_o T$ $B_o T4_o 5_o$ $B_o T5_o 4$
$5_o 4_o B_o T$ $5_o 4_o TB$ $5_o TB_o 4$ $5_o T4_o B$ $5_o B_o 4_o T$ $5_o B_o T4$
$T4_o B_o 5_o$ $T4_o 5_o B$ $T5_o B_o 4$ $T5_o 4_o B$ $TB_o 4_o 5_o$ $TB_o 5_o 4$
KT45: $T_c 4_c 5_c$ $T_o 4_o 5_o$ $T_o 5_o 4$ $4_o T_o 5_o$ $4_o 5_o T$ $5_o T_o 4$ $5_o 4_o T$
KD4B5: $D_c 4_c B_c 5$ $D_o 4_o B_o 5_o$ $D_o 4_o 5_o B$ $D_o 5_o B_o 4$ $D_o 5_o 4_o B$ $D_o B_o 4_o 5$ $D_o B_o 5_o 4$
$4_o D_o B_o 5_o$ $4_o D_o 5_o B$ $4_o 5_o B_o D$ $4_o 5_o D_o B$ $4_o B_o D_o 5_o$ $4_o B_o 5_o D$
$B_o 4_o D_o 5_o$ $B_o 4_o 5_o D$ $B_o 5_o D_o 4$ $B_o 5_o 4_o D$ $B_o D_o 4_o 5_o$ $B_o D_o 5_o 4$
$5_o 4_o B_o D$ $5_o 4_o D_o B$ $5_o D_o B_o 4$ $5_o D_o 4_o B$ $5_o B_o 4_o D$ $5_o B_o D_o 4$
KDB5: $D_c 5_c B_c$ $D_o 5_o B$ ~~$D_o B_o 5_o$~~ $5_o D_o B$ $5_o B_o D$ ~~$B_o 5_o D$~~ ~~$B_o D_o 5_o$~~

Counter-examples were identified for some formulations for the following formulae (formulae 1-117 were screened) :

T; 4; 5; 4^2; 4^3; aiml02_prop3iii; amai02; amai02b; bp,dddnp,g; bp,ddnp,g; bq,dddddddnq; bq,dddddnq; ddddnq,b(q,g); ddddnq,bq;dddnq,bq; ddnq,bq; demri1; demri2; dnq,dbq

Mean execution times for local satisfiability over all the problems 1-117 for the various formulations of S5 that were tested are shown below. The formulations that have a strike-through notation are known to be incomplete (since counter-examples were found). The data is sorted by execution time to illustrate the faster formulations. In order to avoid bias, the data *includes* time points for problems that failed to produce a result before the SPASS time out was reached (these are given a 200 seconds execution time; the real execution time must be greater than 200 seconds), and hence the mean execution times are often under-estimates of the true values.

S5 Formulation	Mean execution Time (msec)	S5 Formulation	Mean execution time (msec)	S5 Formulation	Mean execution time (msec)
~~T4B~~	~~33~~	5oDoB	4120	DoBo5o4	7147
4oTB	70	4oBoD	4125	Bo4o5oT	7156
T4oB	76	4oT5o	4193	Bo4oT5o	7443
~~D4B~~	~~99~~	T4o5o	4218	TBo4o5o	7471
~~BoT4~~	~~109~~	4o5oTB	4240	~~Bo5oD~~	~~7643~~
~~TBo4~~	~~131~~	~~Bo4oD~~	~~4263~~	4o5oBoD	7671
5oT4	217	5oBoD	4678	BoT4o5o	7693
5oT	252	4o5oDoB	4685	5o4oBoD	7819
T5o4	266	T5oBo4	4749	~~DoBo5o~~	~~7871~~
~~T5o~~	~~401~~	T5o4oB	5086	4oDo5oB	7894
5oTB	779	4o5oBoT	5139	Do5oBo4	8236
~~Bo4oT~~	~~1009~~	4oBo5oT	5343	4oBo5oD	8499
4oBoT	1094	5oDo4oB	5352	Do5o4oB	8877
T5oB	1220	4oT5oB	5403	Bo4o5oD	9670
5o4oT	1687	T4o5oB	5478	~~BoDo5o~~	~~10206~~
DoBo4	2603	5oDoBo4	5676	4oBoDo5o	11411
BoT5o4	2701	5o4oDoB	5680	4oDoBo5o	11840
Bo5oT4	2725	Bo5oDo4	5737	Bo4oDo5o	12507
5oBoT	2760	BoDo5o4	5964	Do4o5oB	14006
Bo5oT	2831	Do5oB	5967	BoDo4o5o	18143
5oBoT4	2915	5oBo4oT	6199	Do4oBo5o	18825
~~BoDo4~~	~~2992~~	~~TBo5o~~	~~6231~~	DoBo4o5o	22553
Do4oB	3009	Bo5o4oT	6293	Tc5c	64338
BoT5o	3124	4oBoT5o	6308	TcBc5c	72051
TBo5o4	3145	5oBoDo4	6347	Tc4cBc5c	78390
5o4oTB	3575	5o4oBoT	6645	Tc4cBc	82768
5oT4oB	3593	4oTBo5o	6816	Tc4c5c	92894
4oDoB	3910	T4oBo5o	6859	Dc4cBc	104739
5oTBo4	4060	Bo5o4oD	6903	Dc4cBc5	105320
4o5oT	4087	5oBo4oD	6958	Dc5cBc	106087

Figure 7.12 **Execution times for the *eml-translation to first-order logic* component of the axiomatic translation for all the examples in this study:**

Execution times in SPASS are reported in the output file. These are approximately equal to the CPU execution times (plus the disk access times for input/output). The total execution time, and the execution times of various modules are reported. Over *all* the test examples, with all unique cases of applied axioms and axiom combinations considered (25642 cases) the execution times for the *EML to FOL translation* ranges up to a maximum of 1.06 seconds, with the following statistics.

< 0.01 seconds	56.4 %
0.01-0.1 seconds	41 %
0.1-1.06 seconds	2.6%

Figure 7.13 **Comparing execution times of classical and axiomatic schema translations of modal axioms in local satisfiability calculations:** The table shows the number of test examples (from the formulae 1-119 in the test set) for which execution times of the local satisfiability calculation in classical translation is, the same as, greater than, and lower than the execution time of the axiomatic schema translation. The data is also subdivided according to whether the outcome of the calculation in SPASS was Proof Found (unsatisfiable) or Completion Found (satisfiable). The mean ratios of execution times for axiomatic schema and classical translations are given. The numbers of test cases where formulae were solved in the axiomatic translation, but not in the classical

translation, within the cutoff time of 200 seconds, is also reported. Table cells in which there is no data available, either because the calculation ran out of time for the all cases of the classical translation, or in all cases neither calculation produced a result, are empty. Only a single set of data was analyzed for these results; see the caption of figure 7.5 for more details.

	Execution time classical > axiomatic						Execution time classical < axiomatic					
Axiom	**Complete**		**Total**		**Proof**		**Complete**		**Total**		**Proof**	
	No.	**Mean ratio**	**No.**	**Mean ratio**	**No.**	**Mean ratio**	**No.**	**Mean ratio**	**No.**	**Mean ratio**	**No.**	**Mean ratio**
T	–	–	1	0.40	1	0.40	101	2.4	114	2.5	13	2.7
B	60	0.60	70	0.61	10	0.68	4	1.1	5	1.4	1	3
D	2	0.20	2	0.20	–	–	95	2.8	98	2.8	3	2.2
4	–	–	18	0.21	18	0.21	–	–	–	–	–	–
5_o	–	–	28	0.26	28	0.26	–	–	7	1.5	7	1.5
Alt_1	–	–	14	0.49	14	0.49	–	–	1	2	1	2
4^2	–	–	10	0.11	10	0.11	–	–	–	–	–	–
4^3	–	–	4	0.11	4	0.11	–	–	–	–	–	–
5^2	–	–	12	0.031	12	0.030	–	–	2	2.3	2	2.3
5^3	–	–	2	0.0011	2	0.0011	–	–	2	3.6	2	3.6
$Alt_1^{1,1}$	–	–	7	0.055	7	0.055	–	–	2	2	2	2
$Alt_1^{2,1}$	–	–	5	0.090	5	0.090	–	–	2	2	1	2
$Alt_1^{1,2}$	–	–	2	0.15	2	0.15	–	–	1	2	1	2
$Alt_1^{2,2}$	–	–	1	0.47	1	0.47	–	–	1	2	1	2
T4	–	–	22	0.10	22	0.10	–	–	1	2	1	2
TB	50	0.58	61	0.58	11	0.58	6	1.3	6	1.3	–	–
DB	–	–	–	–	–	–	64	2.8	80	2.8	16	2.8
D4	–	–	22	0.031	22	0.031	14	4.7	15	4.6	1	2
4_oB	–	–	47	0.25	47	0.25	–	–	11	2.1	11	2.1
5_oB	–	–	40	0.33	40	0.33	–	–	14	2.1	14	2.1
5_oT	–	–	58	0.36	58	0.36	–	–	5	1.7	5	1.7
T4B	–	–	38	0.17	38	0.17	–	–	13	1.9	13	1.9
D_o4_oB	–	–	33	0.15	33	0.15	–	–	22	2.8	22	2.8

	Execution time classical = axiomatic			Solved in axiomatic, but not in classical		
Axiom	**Complete No.**	**Total No.**	**Proof No.**	**Complete No.**	**Total No.**	**Proof No.**
T	3	4	1	–	–	–
B	34	44	10	–	–	–
D	14	19	5	–	–	–
4	–	7	7	93	94	1
5_o	–	13	13	66	71	5
Alt_1	–	3	3	100	101	1
4^2	–	2	2	106	107	1
4^3	–	2	2	84	88	4
5^2	–	1	1	47	59	12
5^3	–	1	1	6	18	12
$Alt_1^{1,1}$	–	1	1	78	81	3
$Alt_1^{2,1}$	–	1	1	55	78	23
$Alt_1^{1,2}$	–	1	1	43	63	20
$Alt_1^{2,2}$	–	1	1	19	24	5
T4	–	13	13	77	83	6
TB	33	52	19	–	–	–
DB	30	39	9	–	–	–
D4	–	5	5	72	77	5
4_oB	–	11	11	32	50	18
5_oB	–	17	17	32	47	15
5_oT	–	14	14	26	42	16
T4B	–	17	17	26	51	25
D_o4_oB	–	2	2	23	59	36

Figure 7.14 **<u>Scott-Lemmon translation (correspondence properties) for selected modal-axioms</u>.** The result given is for translation of the formula 5 (¬(→(¬□¬□p,□p))) in each axiom (using 500 seconds execution time; C = Completion Found (Satisfiable); P = Proof Found (Unsatisfiable); X = ran out of execution time) in local satisfiability calculations. The SLF-index is the value of the exponents h, i, j, k in $\Diamond^h\Box^i p\rightarrow\Box^j\Diamond^k p$. Some of the axioms correspond to well-known modal axioms that are described elsewhere in this study, and are named in the second column. The actual formulae generated by the eml-module are reported.

Several formulations give rise to the same axiom under the Scott-Lemmon algorithm – for example those marked **.

SLF index	Axiom Name	Modal Axiom	Correspondence Property Term created within extended SPASS	Result
0000	None	$\Diamond^0\Box^0 p\rightarrow\Box^0\Diamond^0 p$	TRUE	C
1000		$\Diamond^1\Box^0 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V](R_r(V,W)\rightarrow(W\approx V))$	P
0100	T	$\Diamond^0\Box^1 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V]((W\approx V)\rightarrow R_r(W,V))$	C
0010		$\Diamond^0\Box^0 p\rightarrow\Box^1\Diamond^0 p$	$\forall[W,V](R_r(V,W)\rightarrow(W\approx V))$	P
0001		$\Diamond^0\Box^0 p\rightarrow\Box^0\Diamond^1 p$	$\forall[W,V]((W\approx V)\rightarrow R_r(W,V))$	C
1100		$\Diamond^1\Box^1 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V](R_r(V,W)\rightarrow R_r(W,V))$**	C
1010	Alt_1	$\Diamond^1\Box^0 p\rightarrow\Box^1\Diamond^0 p$	$\forall[V,W,U]((R_r(U,V)\wedge R_r(U,W))\rightarrow(W\approx V))$	X
1001		$\Diamond^1\Box^0 p\rightarrow\Box^0\Diamond^1 p$	$\forall[W,V](R_r(V,W)\rightarrow R_r(W,V))$**	C
0110		$\Diamond^0\Box^1 p\rightarrow\Box^1\Diamond^0 p$	$\forall[W,V](R_r(V,W)\rightarrow R_r(W,V))$**	C
0101	D	$\Diamond^0\Box^1 p\rightarrow\Box^0\Diamond^1 p$	$\forall[W,V]((W\approx V)\rightarrow\exists[X](R_r(V,X)\wedge R_r(W,X)))$	C
0011	B	$\Diamond^0\Box^0 p\rightarrow\Box^1\Diamond^1 p$	$\forall[W,V](R_r(V,W)\rightarrow R_r(W,V))$**	C
0111		$\Diamond^0\Box^1 p\rightarrow\Box^1\Diamond^1 p$	$\forall[W,V](R_r(V,W)\rightarrow\exists[X](R_r(V,X)\wedge R_r(W,X)))$	C
1101		$\Diamond^1\Box^1 p\rightarrow\Box^0\Diamond^1 p$	$\forall[W,V](R_r(V,W)\rightarrow\exists[X](R_r(V,X)\wedge R_r(W,X)))$	C
1011	5	$\Diamond^1\Box^0 p\rightarrow\Box^1\Diamond^1 p$	$\forall[V,W,U]((R_r(U,V)\wedge R_r(U,W))\rightarrow R_r(W,V))$	P
1110		$\Diamond^1\Box^1 p\rightarrow\Box^1\Diamond^0 p$	$\forall[V,W,U]((R_r(U,V)\wedge R_r(U,W))\rightarrow R_r(W,V))$	P
1111	G	$\Diamond^1\Box^1 p\rightarrow\Box^1\Diamond^1 p$	$\forall[V,W,U]((R_r(U,V)\wedge R_r(U,W))\rightarrow\exists[X](R_r(V,X)\wedge R_r(W,X)))$	X
0120	4	$\Diamond^0\Box^1 p\rightarrow\Box^2\Diamond^0 p$	$\forall[V,W,Y]((R_r(Y,W)\wedge R_r(V,Y))\rightarrow R_r(W,V))$	X
0210	DEN	$\Diamond^0\Box^2 p\rightarrow\Box^1\Diamond^0 p$	$\forall[Y,W,V](R_r(V,W)\rightarrow(R_r(Y,V)\wedge R_r(W,Y)))$	P
1002		$\Diamond^1\Box^0 p\rightarrow\Box^0\Diamond^2 p$	$\forall[Y,W,V](R_r(V,W)\rightarrow(R_r(Y,V)\wedge R_r(W,Y)))$	P
1120	DBBB	$\Diamond^1\Box^1 p\rightarrow\Box^2\Diamond^0 p$	$\forall[V,U,W,Y]((R_r(U,V)\wedge(R_r(Y,W)\wedge R_r(U,Y)))\rightarrow R_r(W,V))$	X
2100	B^2	$\Diamond^2\Box^1 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V,Y]((R_r(Y,V)\wedge R_r(W,Y))\rightarrow R_r(W,V))$	X
0002		$\Diamond^0\Box^0 p\rightarrow\Box^0\Diamond^2 p$	$\forall[Y,W,V]((W\approx V)\rightarrow(R_r(Y,V)\wedge R_r(W,Y)))$	P
0020		$\Diamond^0\Box^0 p\rightarrow\Box^2\Diamond^0 p$	$\forall[V,W,Y]((R_r(Y,W)\wedge R_r(V,Y))\rightarrow(W\approx V))$	X
2200		$\Diamond^2\Box^2 p\rightarrow\Box^0\Diamond^0 p$	$\forall[Z,W,V,Y]((R_r(Y,V)\wedge R_r(W,Y))\rightarrow(R_r(Z,V)\wedge R_r(W,Z)))$	C
2220		$\Diamond^2\Box^2 p\rightarrow\Box^2\Diamond^0 p$	$\forall[X_1,V,Z,U,W,Y](((R_r(Z,V)\wedge R_r(U,Z))\wedge(R_r(Y,W)\wedge R_r(U,Y)))\rightarrow (R_r(X_1,V)\wedge R_r(W,X_1)))$	C
2020		$\Diamond^2\Box^0 p\rightarrow\Box^2\Diamond^0 p$	$\forall[V,Z,U,W,Y](((R_r(Z,V)\wedge R_r(U,Z))\wedge(R_r(Y,W)\wedge R_r(U,Y)))\rightarrow (W\approx V))$	X
0202		$\Diamond^0\Box^2 p\rightarrow\Box^0\Diamond^2 p$	$\forall[Z,Y,W,V]((W\approx V)\rightarrow \exists[X]((R_r(Z,X)\wedge R_r(V,Z))\wedge(R_r(Y,X)\wedge R_r(W,Y))))$	P
2022		$\Diamond^2\Box^0 p\rightarrow\Box^2\Diamond^2 p$	$\forall[X_1,V,Z,U,W,Y](((R_r(Z,V)\wedge R_r(U,Z))\wedge(R_r(Y,W)\wedge R_r(U,Y)))\rightarrow (R_r(X_1,V)\wedge R_r(W,X_1)))$	C
2202		$\Diamond^2\Box^2 p\rightarrow\Box^0\Diamond^2 p$	$\forall[X_1,Z,W,V,Y]((R_r(Y,V)\wedge R_r(W,Y))\rightarrow \exists[X]((R_r(X_1,X)\wedge R_r(V,X_1))\wedge(R_r(Z,X)\wedge R_r(W,Z))))$	C
2000		$\Diamond^2\Box^0 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V,Y]((R_r(Y,V)\wedge R_r(W,Y))\rightarrow(W\approx V))$	X
2222		$\Diamond^2\Box^2 p\rightarrow\Box^2\Diamond^2 p$	$\forall[X_2,X_1,V,Z,U,W,Y](((R_r(Z,V)\wedge R_r(U,Z))\wedge(R_r(Y,W)\wedge R_r(U,Y)))\rightarrow \exists[X]((R_r(X_2,X)\wedge R_r(V,X_2))\wedge(R_r(X_1,X)\wedge R_r(W,X_1))))$	C
0022		$\Diamond^0\Box^0 p\rightarrow\Box^2\Diamond^2 p$	$\forall[Z,V,W,Y]((R_r(Y,W)\wedge R_r(V,Y))\rightarrow(R_r(Z,V)\wedge R_r(W,Z)))$	C
0222		$\Diamond^0\Box^2 p\rightarrow\Box^2\Diamond^2 p$	$\forall[X_1,Z,V,W,Y]((R_r(Y,W)\wedge R_r(V,Y))\rightarrow \exists[X]((R_r(X_1,X)\wedge R_r(V,X_1))\wedge(R_r(Z,X)\wedge R_r(W,Z)))$	C
3333		$\Diamond^3\Box^3 p\rightarrow\Box^3\Diamond^3 p$	$\forall[X_6,X_5,X_4,X_3,V,X_2,X_1,U,W,Z,Y](((R_r(X_2,V)\wedge(R_r(X_1,X_2)\wedge R_r(U,X_1)))\wedge(R_r(Z,W)\wedge(R_r(Y,Z)\wedge R_r(U,Y))))\rightarrow \exists[X]((R_r(X_6,X)\wedge(R_r(X_5,X_6)\wedge R_r(V,X_5)))\wedge(R_r(X_4,X) \wedge(R_r(X_3,X_4)\wedge R_r(W,X_3)))))$	C
3100	B^3	$\Diamond^3\Box^1 p\rightarrow\Box^0\Diamond^0 p$	$\forall[W,V,Z,Y]((R_r(Z,V)\wedge(R_r(Y,Z)\wedge R_r(W,Y)))\rightarrow R_r(W,V))$	X

Figure 7.15 <u>Summary of the correspondence properties and axiomatic schema of some modal axioms</u>. The correspondence properties are taken from [26]. Similar properties are seen in sections 2.2.2 and 2.2.3, and figures 2.8 and 2.18, taken from [1]. See these sections for an explanation. Briefly, new symbols arise from the schema encoding of the axiom, and need to be Defined; compositional terms are added to the current instantiation set. Axioms CR2/CR3 were previously implemented in `ml2dfg` (described in [1]) as axioms A/P; axioms B^2 and M were also implemented in `ml2dfg`.

Axiom	Correspondence property ***	**AxiomaticTranslation**		
		Schema encoding	**New Symbols**	**Composition Terms**
Shift Reflexive (SR) $\Box(\Box p\to p)$	$R(x,y) \to R(y,y)$	$\forall x\forall y(\neg R(x,y)\vee\neg Q_{\Box p}(y)\vee Q_p(y))$	None	$\Box(\Box p\to p)$
Convergence or confluence (G)* $\Diamond\Box p\to\Box\Diamond p$	$R(y,z)\wedge R(y,u) \to \exists x(R(z,x)\wedge R(u,x))$	$\forall x(\forall y(\neg R(x,y)\vee\neg Q_{\Box p}(y)) \vee \forall y(\neg R(x,y)\vee\neg Q_{\Box\neg p}(y))$	$\Box\neg p$	$\Box\neg\Box p$, $\Box\neg\Box\neg p$
Dense (Den)** $\Box\Box p\to\Box p$	$R(y,z) \to \exists x(R(x,z)\wedge R(y,x))$	$\forall x(\exists y(R(x,y)\wedge\neg Q_{\Box p}(y))\vee Q_{\Box p}(x))$	None	$\Box\Box p$
Trivial (Tr) $\Box p\leftrightarrow p$	$R(x,y)\leftrightarrow x\approx y$	$\forall x((\neg Q_{\Box p}(x)\vee Q_p(x))\wedge(\neg Q_p(x)\vee Q_{\Box p}(x)))$	None	None
McKinsey (M) $\Box\Diamond p\to\Diamond\Box p$	No first order correspondence property	$\forall x(\exists y(R(x,y)\wedge Q_{\Box\neg p}(y)) \vee \exists y(R(x,y)\wedge Q_{\Box p}(y)))$	$\Box\neg p$	$\Box\neg\Box p$, $\Box\neg\Box\neg p$
Löb (W) $\Box(\Box p\to p)\to\Box p$	No first order correspondence property	$\forall x(\exists y(R(x,y)\wedge Q_{\Box p}(y)\wedge\neg Q_p(y))\vee Q_{\Box p}(x))$	None	$\Box(\Box p\to p)$
CR $[r]p \to [s]p$		$\forall x(\neg Q_{[r]p}(x) \vee Q_{[s]p}(x))$	None	None
CR2 $[r]p \to [s][r]p$	$(R_s(x,y) \wedge R_r(y,z)) \to R_r(x,z)$	$\forall x(\neg Q_{[r]p}(x) \vee \forall y(\neg R_s(x,y) \vee Q_{[r]p}(y)))$	None	$[s][r]p$
CR3 $[r]p \to [r][s]p$	$(R_r(x,y) \wedge R_s(y,z)) \to R_r(x,z)$	$\forall x(\neg Q_{[r]p}(x) \vee \forall y(\neg R_r(x,y) \vee Q_{[s]p}(y)))$	None	$[r][s]p$
B^2 $\Diamond\Diamond\Box p \to p$	$(R(x,y) \wedge R(y,z)) \to R(z,x)$	$\forall x(\forall y(\neg R(x,y)\vee \forall z(\neg R(y,z) \vee \neg Q_{\Box p}(z))) \vee Q_p(x))$	None	$\Box\Box\neg\Box p$, $\Box\neg\Box p$
B^3 $\Diamond\Diamond\Diamond\Box p \to p$	$(R(z,v)\wedge R(y,z)\wedge R(w,y))\to R(w,v)$	$\forall x(\forall y(\neg R(x,y) \vee \forall z(\neg R(y,z) \vee \forall u(\neg R(z,u)\vee \neg Q_{\Box p}(u)))) \vee Q_p(x))$	None	$\Box\Box\Box\neg\Box p$, $\Box\Box\neg\Box p$, $\Box\neg\Box p$
DBBB $\Diamond\Box p \to \Box\Box p$	$(R(x,y)\wedge R(x,u)\wedge R(u,z)) \to R(y,z)$	$\forall x(\forall y(\neg R(x,y)\vee \neg Q_{\Box p}(y)) \vee \forall y(\neg R(x,y) \vee Q_{\Box p}(y)))$	None	$\Box\neg\Box p$, $\Box\Box p$

*In Scott-Lemmon with h=i=j=k=1. ** In Scott-Lemmon with h=k=0, i=2, j=1.

***If not otherwise stated all variables are universally quantified.

Figure 7.16 <u>Derivation of the axiomatic schema of some modal axioms</u>. The derivation of the axiomatic schema encodings from figure 7.15 is shown below. These axioms were not presented in [1], and are used here to illustrate features of the axiomatic translation and the implementation of the translation in extended-SPASS. It should be noted that in all cases it is possible to derive alternate formulations of the modal axioms that would lead to different formulations of the axiomatic translations.

$\forall p\forall x(\pi(\neg p,x) \leftrightarrow \neg\pi(p, x))$ (1)
$\forall pq\forall x(\pi(p*q,x) \leftrightarrow (\pi(p, x)*\pi(q, x)))$ (2) where $* \in \{\to,\leftrightarrow,\vee,\wedge\}$
$\forall p\forall x(\pi(\Box p,x) \leftrightarrow \forall y(R(x,y) \to \pi(p, y)))$ (3) [again, formulae 2.4]

Axiom SR: $\Box(\Box p\to p)$

$\forall p\forall x(\pi(\Box(\Box p\to p),x))$ = $\forall p\forall x(\forall y(R(x,y) \to \pi(\Box p\to p,x)))$
= $\forall p\forall x(\forall y(R(x,y) \to (\pi(\Box p,y) \to \pi(p,y))))$
Re-writing gives $\forall x(\forall y(R(x,y) \to (Q_{\Box p}(y) \to Q_p(y))))$
= $\mathbf{\forall x\forall y(\neg R(x,y) \vee \neg Q_{\Box p}(y) \vee Q_p(y))}$

Axiom G: $\Diamond\Box p\to\Box\Diamond p$

$\forall p\forall x(\pi(\Diamond\Box p\to\Box\Diamond p,x))$ = $\forall p\forall x(\pi(\Diamond\Box p,x)\to\pi(\Box\Diamond p,x))$

= $\forall p\forall x(\pi(\neg\Box\neg\Box p,x)\to\pi(\Box\neg\Box\neg p,x))$

= $\forall p\forall x(\neg(\forall y(R(x,y)\to\pi(\neg\Box p,y)))\to(\forall y(R(x,y)\to\pi(\neg\Box\neg p,y)))$

= $\forall p\forall x(\neg(\forall y(R(x,y)\to\neg\pi(\Box p,y)))\to(\forall y(R(x,y)\to\neg\pi(\Box\neg p,y)))$

Re-writing gives $\forall x(\neg(\forall y(R(x,y)\to\neg Q_{\Box p}(y)))\to(\forall y(R(x,y)\to\neg Q_{\Box\neg p}(y)))$

= $\mathbf{\forall x(\forall y(\neg R(x,y)\vee\neg Q_{\Box p}(y))\vee\forall y(\neg R(x,y)\vee\neg Q_{\Box\neg p}(y))}$

Axiom Dense: $\Box\Box p\to\Box p$

$\forall p\forall x(\pi(\Box\Box p\to\Box p,x))$ = $\forall p\forall x(\pi(\Box\Box p,x)\to\pi(\Box p,x))$

= $\forall p\forall x(\forall y(R(x,y)\to\pi(\Box p,y))\to\pi(\Box p,x))$

Re-writing gives $\forall x(\forall y(R(x,y)\to Q_{\Box p}(y))\to Q_{\Box p}(x))$

= $\forall x(\neg\forall y(\neg R(x,y)\vee Q_{\Box p}(y))\vee Q_{\Box p}(x))$

= $\mathbf{\forall x(\exists y(R(x,y)\wedge\neg Q_{\Box p}(y))\vee Q_{\Box p}(x))}$

Axiom Trivial: $\Box p\leftrightarrow p$

$\forall p\forall x(\pi(\Box p\leftrightarrow p,x))$ ≡ $\forall p\forall x(\pi(\Box p,x)\leftrightarrow\pi(p,x))$

Re-writing gives $\forall x(Q_{\Box p}(x)\leftrightarrow Q_p(x))$

= $\forall x((Q_{\Box p}(x)\to Q_p(x))\wedge(Q_p(x)\to Q_{\Box p}(x)))$

= $\mathbf{\forall x((\neg Q_{\Box p}(x)\vee Q_p(x))\wedge(\neg Q_p(x)\vee Q_{\Box p}(x)))}$

Axiom M: $\Box\Diamond p\to\Diamond\Box p$

$\forall p\forall x(\pi(\Box\Diamond p\to\Diamond\Box p,x))$ = $\forall p\forall x(\pi(\Box\Diamond p,x)\to\pi(\Diamond\Box p,x))$

= $\forall p\forall x(\pi(\Box\neg\Box\neg p,x)\to\pi(\neg\Box\neg\Box p,x))$

= $\forall p\forall x(\forall y(R(x,y)\to\pi(\neg\Box\neg p,y))\to\neg\pi(\Box\neg\Box p,x))$

= $\forall p\forall x(\forall y(R(x,y)\to\neg\pi(\Box\neg p,y))\to\neg\forall y(R(x,y)\to\neg\pi(\Box p,y)))$

Re-writing gives $\forall x(\forall y(R(x,y)\to\neg Q_{\Box\neg p}(y))\to\neg\forall y(R(x,y)\to\neg Q_{\Box p}(y)))$

= $\forall x(\neg\forall y(\neg R(x,y)\vee\neg Q_{\Box\neg p}(y))\vee\neg\forall y(\neg R(x,y)\vee\neg Q_{\Box p}(y)))$

= $\mathbf{\forall x(\exists y(R(x,y)\wedge Q_{\Box\neg p}(y))\vee\exists y(R(x,y)\wedge Q_{\Box p}(y)))}$

Axiom W: $\Box(\Box p\to p)\to\Box p$

$\forall p\forall x(\pi(\Box(\Box p\to p)\to\Box p,x))$ = $\forall p\forall x(\pi(\Box(\Box p\to p),x)\to\pi(\Box p,x))$

= $\forall p\forall x(\forall y(R(x,y)\to\pi(\Box p\to p,y))\to\pi(\Box p,x))$

= $\forall p\forall x(\forall y(R(x,y)\to(\pi(\Box p,y)\to\pi(p,y)))\to\pi(\Box p,x))$

Re-writing gives $\forall x(\forall y(R(x,y)\to(Q_{\Box p}(y)\to Q_p(y)))\to Q_{\Box p}(x))$

= $\forall x(\neg\forall y(\neg R(x,y)\vee\neg Q_{\Box p}(y)\vee Q_p(y))\vee Q_{\Box p}(x))$

= $\mathbf{\forall x(\exists y(R(x,y)\wedge Q_{\Box p}(y)\wedge\neg Q_p(y))\vee Q_{\Box p}(x))}$

Axiom CR: $[R]p\to[S]p$

$\forall p\forall x(\pi([R]p\to[S]p,x))$ = $\forall p\forall x(\pi([R]p,x)\to\pi([S]p,x))$

Re-writing gives $\forall x(Q_{[R]p}(x)\to Q_{[S]p}(x))$

= $\forall x(\neg Q_{[R]p}(x)\vee Q_{[S]p}(x))$

Axiom CR2: $[R]p\to[S][R]p$

$\forall p\forall x(\pi([R]p\to[S][R]p,x))$ = $\forall p\forall x(\pi([R]p,x)\to\pi([S][R]p,x))$

= $\forall p\forall x(\pi([R]p,x)\to\forall y(S(x,y)\to\pi([R]p,y)))$

Re-writing gives $\forall x(Q_{[R]p}(x)\to\forall y(S(x,y)\to Q_{[R]p}(y)))$

= $\forall x(\neg Q_{[R]p}(x)\vee\forall y(\neg S(x,y)\vee Q_{[R]p}(y)))$

Axiom CR3: $[R]p\to[R][S]p$

$\forall p\forall x(\pi([R]p\to[R][S]p,x))$ = $\forall p\forall x(\pi([R]p,x)\to\pi([R][S]p,x))$

= $\forall p\forall x(\pi([R]p,x)\to\forall y(R(x,y)\to\pi([S]p,y)))$

Re-writing gives $\forall x(Q_{[R]p}(x)\to\forall y(R(x,y)\to Q_{[S]p}(y)))$

= $\forall x(\neg Q_{[R]p}(x)\vee\forall y(\neg R(x,y)\vee Q_{[S]p}(y)))$

Axiom B^2: $\Diamond\Diamond\Box p\to p$

$\forall p\forall x(\pi(\neg\Box\Box\neg\Box p\to p,x))$ ≡ $\forall p\forall x(\pi(\neg\Box\Box\neg\Box p,x)\to\pi(p,x))$

≡ $\forall p\forall x(\neg\pi(\Box\Box\neg\Box p,x)\to\pi(p,x))$

≡ $\forall p\forall x(\neg\forall y(R(x,y)\to\pi(\Box\neg\Box p,y))\to\pi(p,x))$

≡ $\forall p\forall x(\neg\forall y(R(x,y)\to\pi(\Box\neg\Box p,y))\to\pi(p,x))$

≡ $\forall p\forall x(\neg\forall y(R(x,y)\to\forall z(R(y,z)\to\pi(\neg\Box p,z)))\to\pi(p,x))$

≡ $\forall p\forall x(\neg\forall y(R(x,y)\to\forall z(R(y,z)\to\neg\pi(\Box p,z)))\to\pi(p,x))$

Re-writing gives $\forall x(\neg\forall y(R(x,y)\to\forall z(R(y,z)\to\neg Q_{\Box p}(z)))\to Q_p(x))$

≡ $\mathbf{\forall x(\forall y(\neg R(x,y)\vee\forall z(\neg R(y,z)\vee\neg Q_{\Box p}(z)))\vee Q_p(x))}$

Axiom B^3: $\Diamond\Diamond\Diamond\Box p\to p$

$\forall p\forall x(\pi(\neg\Box\Box\Box\neg\Box p\to p,x))$ ≡ $\forall p\forall x(\pi(\neg\Box\Box\Box\neg\Box p,x)\to\pi(p,x))$

≡ $\forall p\forall x(\neg\pi(\Box\Box\Box\neg\Box p,x)\to\pi(p,x))$

	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \pi(\Box\Box\neg\Box p,y)) \rightarrow \pi(p,x))$
	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \forall z(R(y,z) \rightarrow \pi(\Box\neg\Box p,z))) \rightarrow \pi(p,x))$
	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \forall z(R(y,z) \rightarrow \forall u(R(z,u) \rightarrow \pi(\neg\Box p,u)))) \rightarrow \pi(p,x))$
	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \forall z(R(y,z) \rightarrow \forall u(R(z,u) \rightarrow \neg\pi(\Box p,u)))) \rightarrow \pi(p,x))$
Re-writing gives		$\forall x(\neg \forall y(R(x,y) \rightarrow \forall z(R(y,z) \rightarrow \forall u(R(z,u) \rightarrow \neg Q_{\Box p}(u)))) \rightarrow Q_p(x))$
	≡	$\mathbf{\forall x(\forall y(\neg R(x,y) \vee \forall z(\neg R(y,z) \vee \forall u(\neg R(z,u) \vee \neg Q_{\Box p}(u)))) \vee Q_p(x))}$
Axiom DBBB: $\Diamond\Box p \rightarrow \Box\Box p$		
$\forall p \forall x(\pi(\neg\Box\neg\Box p \rightarrow \Box\Box p,x))$	≡	$\forall p \forall x(\pi(\neg\Box\neg\Box p,x) \rightarrow \pi(\Box\Box p,x))$
	≡	$\forall p \forall x(\neg\pi(\Box\neg\Box p,x) \rightarrow \forall y(R(x,y) \rightarrow \pi(\Box p,y)))$
	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \pi(\neg\Box p,y)) \rightarrow \forall y(R(x,y) \rightarrow \pi(\Box p,y)))$
	≡	$\forall p \forall x(\neg \forall y(R(x,y) \rightarrow \neg\pi(\Box p,y)) \rightarrow \forall y(R(x,y) \rightarrow \pi(\Box p,y)))$
Re-writing gives		$\forall x(\neg \forall y(R(x,y) \rightarrow \neg Q_{\Box p}(y)) \rightarrow \forall y(R(x,y) \rightarrow Q_{\Box p}(y)))$
	≡	$\forall x(\forall y(\neg R(x,y) \vee \neg Q_{\Box p}(y)) \vee \forall y(\neg R(x,y) \vee Q_{\Box p}(y)))$

Figure 7.17 Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations. The target formulae are listed in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 4.6 hours. These axioms are not discussed in [1]. A single set of experiments was run. There are no differences in outcome for axiom combinations DDen, D_oDen_o, DenD, Den_oD_o over all the test examples. There are no differences in outcome for the axioms DBBB vs. $DBBB_o$, G vs. G_o, SR vs. SR_o, TR vs. TR_o.

No.	Target formula	DBBB	$DBBB_o$	DDEN	D_oDEN_o	DEND	D_oEND_o	DEN	DEN_o	G	G_o	SR	SR_o	TR	TR_o
1	4	C	C	C	C	C	C	C	C	C	C	C	C	P	P
2	4^2	C	C	C	C	C	C	C	C	C	C	C	C	P	P
3	4^3	C	C	C	C	C	C	C	C	C	C	C	C	P	P
4	5	C	C	C	C	C	C	C	C	C	C	C	C	P	P
5	5^2	C	C	C	C	C	C	C	C	C	C	C	C	P	P
6	5^3	C	C	C	C	C	C	C	C	C	C	C	C	P	P
7	D	C	C	P	P	P	P	C	C	C	C	C	C	P	P
8	T	C	C	C	C	C	C	C	C	C	C	C	C	P	P
9	alt1	C	C	C	C	C	C	C	C	C	C	C	C	P	P
10	$alt_1^{1,1}$	C	C	C	C	C	C	C	C	C	C	C	C	P	P
11	alt1^1,2	C	X	C	C	C	C	C	C	C	C	C	C	P	P
12	alt1^2,1	C	X	C	C	C	C	C	C	C	C	C	C	P	P
13	alt1^2,2	C	X	C	C	C	C	C	C	C	C	C	C	P	P
14	B	C	C	C	C	C	C	C	C	C	C	C	C	P	P
15	B^2	C	C	C	C	C	C	C	C	C	C	C	C	P	P
16	F	C	X	C	C	C	C	C	C	C	X	C	C	P	P
17	M	C	C	C	C	C	C	C	C	C	C	C	C	P	P
18	Cxt	P	P	C	C	C	C	C	C	C	C	C	C	P	P
19	bbp,dddd(q,dnp)	C	X	C	C	C	C	C	C	C	X	C	C	P	P
20	bp,dddnp,g	C	X	C	C	C	C	C	C	C	X	C	C	P	P
21	bp,dddp,g	C	X	C	C	C	C	C	C	C	X	C	C	C	C
22	bp,ddnp,g	C	X	C	C	C	C	C	C	C	X	C	C	P	P
23	bp,ddp	C	C	C	C	C	C	C	C	C	C	C	C	C	C
24	bq,d	C	C	C	C	C	C	C	C	C	C	C	C	C	C
25	bq,ddddddnq	C	X	C	C	C	C	C	C	C	X	C	C	P	P
26	bq,dddddnq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
27	d(bp,ddnq),ddbq	P	P	C	C	C	C	C	C	C	C	C	C	P	P
28	d(dddddnq,ddddbq)	P	P	C	C	C	C	C	C	C	X	C	C	P	P
29	d(ddddnq,dddbq)	P	P	C	C	C	C	C	C	C	X	C	C	P	P
30	d(ddnq,dbq)	P	P	C	C	C	C	C	C	C	C	C	C	P	P
31	d(dnq,dbq)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
32	d(dnq,dddbq)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
33	dbp,dd(np,bq)	P	P	C	C	C	C	C	C	C	X	C	C	P	P
34	dbp,dddddnp	C	X	C	C	C	C	C	C	C	C	C	C	P	P
35	dbp,ddnp	P	P	C	C	C	C	C	C	C	C	C	C	P	P
36	dbq,d(dnq,ddp)	P	P	C	C	C	C	C	C	C	X	C	C	P	P
37	dd(bq,ddddnq)	C	X	C	C	C	C	C	C	C	X	C	C	P	P
38	dd(dnq,dbq)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
39	dddddnq,dp,bbp	C	X	C	C	C	C	C	C	C	X	C	C	P	P

40	ddddddnq,bbbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
41	ddddddnq,ddbq	C	X	C	C	C	C	C	C	C	C	C	C	P	P
42	ddddddnq,dddbq	C	X	C	C	C	C	C	C	C	X	C	C	P	P
43	ddddddnq,ddddbq	P	P	C	C	C	C	C	C	C	X	C	C	P	P
44	ddddnq,b(q,g)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
45	ddddnq,bbp	C	C	C	C	C	C	C	C	C	C	C	C	P	P
46	ddddnq,bq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
47	ddddnq,ddbq	C	X	C	C	C	C	C	C	C	C	C	C	P	P
48	ddddnq,dddbq	P	P	C	C	C	C	C	C	C	X	C	C	P	P
49	ddddnq,dp,bbp	C	C	C	C	C	C	C	C	C	C	C	C	P	P
50	dddnq,bbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
51	dddnq,bq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
52	dddnq,dbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
53	dddnq,ddbq	P	P	C	C	C	C	C	C	C	C	C	C	P	P
54	dddnq,dddbq	C	X	C	C	C	C	C	C	C	C	C	C	P	P
55	dddnq,ddddbq	C	X	C	C	C	C	C	C	C	C	C	C	P	P
56	dddnq,dp,bbp	C	C	C	C	C	C	C	C	C	C	C	C	P	P
57	ddnq,bq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
58	ddnq,dbq	P	P	C	C	C	C	C	C	C	C	C	C	P	P
59	ddnq,ddbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
60	ddnq,dddbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
62	Nnd	C	C	C	C	C	C	C	C	C	C	C	C	C	C
63	dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
64	dnq,dbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
65	dnq,ddbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
66	dnq,dddbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
67	dnq,ddp,bbbq	C	C	P	P	P	P	P	P	C	C	P	P	P	P
68	n(bp,dp)	C	C	P	P	P	P	C	C	C	C	C	C	P	P
69	n(d1d2b3bp->p)	C	X	C	C	C	C	C	C	C	X	C	C	P	P
70	n(dbbbbp->bbdp)	C	X	C	C	C	C	C	C	C	X	C	C	P	P
71	n(dbbp->bp)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
72	n(dbdnp->nbbp)	C	C	C	C	C	C	C	C	C	C	P	P	P	P
73	n(dbp->dp)	C	C	C	C	C	C	C	C	C	C	P	P	P	P
74	n(ddbbp->p)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
75	n(dp->p)	C	C	C	C	C	C	C	C	C	C	C	C	P	P
76	nd,g	C	C	P	P	P	P	C	C	C	C	C	C	P	P
77	nd	C	C	P	P	P	P	C	C	C	C	C	C	P	P
78	np,bbbp,dddddddq	C	X	C	C	C	C	C	C	C	X	C	C	P	P
79	np,bbp,dddddddq	C	X	C	C	C	C	C	C	C	X	C	C	P	P
80	np,ddddddddbp	C	X	C	C	C	C	C	C	C	X	C	C	P	P
81	np,dddddddddbp	C	X	C	C	C	C	C	C	C	X	C	C	C	C
82	nq,ddbq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
83	nq,dddddbq	C	X	C	C	C	C	C	C	C	C	C	C	P	P
84	p,bbndp,ddq	C	C	C	C	C	C	C	C	C	C	C	C	P	P
85	(db)^1p,p	C	C	C	C	C	C	C	C	C	C	C	C	P	P
86	(db)^1p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C
87	(db)^2p,p	C	C	C	C	C	C	C	C	C	C	C	C	P	P
88	(db)^2p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C
89	(db)^3p,p	C	X	C	C	C	C	C	C	C	C	C	C	P	P
90	(db)^3p,q	C	X	C	C	C	C	C	C	C	C	C	C	C	C
105	aiml02_prop3i	C	X	C	C	C	C	C	C	C	X	C	C	P	P
106	aiml02_prop3ii	C	X	C	C	C	C	C	C	C	X	C	C	P	P
107	aiml02_prop3iii	C	X	C	C	C	C	C	C	C	X	C	C	P	P
108	amai02	C	C	C	C	C	C	C	C	C	C	C	C	P	P
109	amai02b	C	C	C	C	C	C	C	C	C	C	C	C	P	P
110	demri1	C	C	C	C	C	C	C	C	C	C	C	C	P	P
111	demri2	C	X	C	C	C	C	C	C	C	X	C	C	P	P
112	demri3	P	P	P	P	P	P	P	P	P	P	P	P	P	P
113	demri5	C	X	C	C	C	C	C	C	C	X	C	C	P	P
114	demri6	C	X	C	C	C	C	C	C	C	X	C	C	C	C
115	demri7	C	X	C	C	C	C	C	C	C	X	C	C	P	P
116	demri8	C	X	C	C	C	C	C	C	C	X	C	C	P	P
117	demri9	C	X	C	C	C	C	C	C	C	X	C	C	C	C

Figure 7.18 Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations. The target formulae are listed in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 29.4 hours (excluding the data for axioms M, M_o, W, and W_o). These axioms are not discussed in [1]. A single set of experiments was

analyzed.

Software testing: It was confirmed that, where a result (Proof or Completion) was produced for both the translation of a particular formula with the axiom translated as correspondence property and axiom translated as an axiomatic schema, that there was *no difference* between these outcomes in any of the cases tested (the outcomes for G_o = the outcomes for G_c, $B^2{}_o = B_c$, etc).

Counter-examples are seen for the outcomes of experiments with axiom B^2 vs. $B^2{}_o$. These counter-examples are listed below:

```
4^3; 5^3; Cxt; bbp,dddd(q,dnp); bq,dddddddnq;
d(bp,ddnq),ddbq; dbp,dd(np,bq); dbp,dddddnp; dbp,ddnp;
dbq,d(dnq,ddp); ddddnq,b(q,g); ddddnq,bq; dddnq,ddbq;
ddnq,dbq; dnq,dddbq; nq,dddddbq; np,ddddddddbp
```

No.	Target Formula	B^2	$B^2{}_o$	B^3	$B^3{}_o$	$B^2{}_c$	$B^3{}_c$	$DBBB_c$	DEN_c	G_c	SR_c	TR_c	M	M_o	W	W_o
1	4	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
2	4^2	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
3	4^3	C	P	C	X	X	X	X	C	X	C	P	C	C	C	C
4	5	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
5	5^2	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
6	5^3	C	P	C	X	P	X	X	C	X	C	P	C	C	C	C
7	D	C	C	C	X	X	X	X	C	X	C	P	P	P	C	C
8	T	C	C	C	C	X	X	X	C	X	C	P	C	C	C	C
9	alt1	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
10	alt1^1,1	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
11	alt1^1,2	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
12	alt1^2,1	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
13	alt1^2,2	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
14	B	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
15	B^2	P	P	C	X	P	X	X	C	X	C	P	C	C	C	C
16	F	C	C	C	X	X	X	X	C	X	C	P	C	X	C	C
17	M	C	C	C	X	X	X	X	C	X	C	P	P	P	C	C
18	Cxt	C	P	C	X	P	X	P	C	X	C	P	C	C	C	C
19	bbp,dddd(q,dnp)	C	P	C	X	X	X	X	C	X	C	P	C	X	C	C
20	bp,dddnp,g	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
21	bp,dddp,g	C	X	X	X	X	X	X	C	X	C	X	C	X	C	C
22	bp,ddnp,g	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
23	bp,ddp	C	C	C	X	X	X	X	C	X	C	X	C	C	C	C
24	bq,d	C	C	C	C	X	X	X	C	X	C	X	C	C	C	C
25	bq,dddddddnq	C	P	C	X	X	X	X	C	X	C	P	C	X	C	C
26	bq,dddddnq	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
27	d(bp,ddnq),ddbq	C	P	C	X	P	X	P	C	X	C	P	C	X	C	C
28	d(dddddnq,ddddbq)	P	P	P	P	P	X	X	C	X	C	P	C	X	C	C
29	d(ddddnq,dddbq)	P	P	P	P	P	X	P	C	X	C	P	C	X	C	C
30	d(ddnq,dbq)	P	P	P	P	P	X	P	C	X	C	P	C	C	C	C
31	d(dnq,dbq)	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
32	d(dnq,dddbq)	P	P	C	X	P	X	X	C	X	C	P	C	X	C	C
33	dbp,dd(np,bq)	C	P	C	X	P	X	P	C	X	C	P	C	X	C	C
34	dbp,dddddnp	C	P	C	X	P	X	X	C	X	C	P	C	C	C	C
35	dbp,ddnp	C	P	C	X	P	X	P	C	X	C	P	C	C	C	C
36	dbq,d(dnq,ddp)	C	P	C	X	P	X	P	C	X	C	P	C	X	C	C
37	dd(bq,ddddnq)	P	P	C	X	P	X	X	C	X	C	P	C	X	C	C
38	dd(dnq,dbq)	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
39	ddddddnq,dp,bbp	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
40	dddddnq,bbbq	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
41	dddddnq,ddbq	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
42	dddddnq,dddbq	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
43	dddddnq,ddddbq	P	P	P	P	P	X	P	C	X	C	P	C	X	C	C
44	ddddnq,b(q,g)	C	P	C	X	X	X	X	C	X	C	P	C	C	C	C
45	ddddnq,bbp	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
46	ddddnq,bq	C	P	C	X	X	X	X	C	X	C	P	C	C	C	C
47	ddddnq,ddbq	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
48	ddddnq,dddbq	P	P	X	X	P	X	P	C	X	C	P	C	X	C	C
49	ddddnq,dp,bbp	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
50	dddnq,bbq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
51	dddnq,bq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
52	dddnq,dbq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C

53	dddnq,ddbq	C	P	C	X	P	X	P	C	X	C	P	C	C	C	C
54	dddnq,dddbq	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
55	dddnq,ddddbq	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
56	dddnq,dp,bbp	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
57	ddnq,bq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
58	ddnq,dbq	C	P	C	X	P	X	P	C	X	C	P	C	C	C	C
59	ddnq,ddbq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
60	ddnq,dddbq	C	X	C	X	X	X	X	C	X	C	P	C	C	C	C
61	ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
62	Nnd	C	C	C	C	X	X	X	C	X	C	C	C	C	C	C
63	dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
64	dnq,dbq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
65	dnq,ddbq	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
66	dnq,dddbq	C	P	C	X	P	X	X	C	X	C	P	C	C	C	C
67	dnq,ddp,bbbq	C	X	C	X	X	X	X	P	X	P	P	C	C	C	C
68	n(bp,dp)	C	C	C	X	X	X	X	C	X	C	P	P	P	C	C
69	n(d1d2b3bp->p)	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
70	n(dbbbbp->bbdp)	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
71	n(dbbp->bp)	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
72	n(dbdnp->nbbp)	C	C	C	X	X	X	X	C	X	P	P	C	C	C	C
73	n(dbp->dp)	C	C	C	X	X	X	X	C	X	P	P	C	C	C	C
74	n(ddbbp->p)	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
75	n(dp->p)	C	C	C	C	X	X	X	C	X	C	P	C	C	C	C
76	nd,g	C	C	C	X	X	X	X	C	X	C	P	P	P	C	C
77	nd	C	C	C	C	X	X	X	C	X	C	P	P	P	C	C
78	np,bbbp,dddddddq	P	P	X	X	P	X	X	C	X	C	P	C	X	C	C
79	np,bbp,dddddddq	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
80	np,ddddddddbp	C	P	X	X	X	X	X	C	X	C	P	C	X	C	C
81	np,dddddddddbp	C	X	X	X	X	X	X	C	X	C	X	C	X	C	C
82	nq,ddbq	P	P	C	X	P	X	X	C	X	C	P	C	C	C	C
83	nq,dddddbq	C	P	C	X	P	X	X	C	X	C	P	C	X	C	C
84	p,bbndp,ddq	P	P	C	X	P	X	X	C	X	C	P	C	C	C	C
85	(db)^1p,p	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
86	(db)^1p,q	C	C	C	X	X	X	X	C	X	C	X	C	C	C	C
87	(db)^2p,p	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
88	(db)^2p,q	C	C	C	X	X	X	X	C	X	C	X	C	C	C	C
89	(db)^3p,p	C	X	X	X	X	X	X	C	X	C	P	C	X	C	C
90	(db)^3p,q	C	X	C	X	X	X	X	C	X	C	X	C	X	C	C
105	aiml02_prop3i	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
106	aiml02_prop3ii	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
107	aiml02_prop3iii	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
108	amai02	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
109	amai02b	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
110	demri1	C	C	C	X	X	X	X	C	X	C	P	C	C	C	C
111	demri2	C	X	C	X	X	X	X	C	X	C	P	C	X	C	C
112	demri3	P	P	P	P	P	P	P	P	P	P	P				
113	demri5	C	X	C	X	X	X	X	C	X	C	P				
114	demri6	C	X	X	X	X	X	X	C	X	C	X				
115	demri7	C	X	X	X	X	X	X	C	X	C	P				
116	demri8	C	X	X	X	X	X	X	C	X	C	P				
117	demri9	C	X	X	X	X	X	X	C	X	C	X				

Figure 7.19 <u>Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations</u>. The data is taken from the results for test problems 1-90 and 105-117 (see figure 7.4 for details). The data analyzed corresponds to that presented in figures 7.17 to 7.18. Problems are deemed to have failed if they were not solved before running out of time at 200 seconds execution time. Execution time statistics are listed for the solved problems. The statistics for all the problems, regardless of outcome, are all listed under the heading *Total*. This set of solved problems is sub-divided by the outcome, either Completion Found in SPASS (satisfiable), or Proof Found in SPASS (unsatisfiable), in the other two sections. Where there is no data (no problems solved) the table is blank. The median, arithmetic mean ± standard deviation, and maximum values are recorded. A single set of experiments was analyzed.

	Total set of problems				Problem outcome = Proof				Problem outcome = Completion			
Axiom	No. Problems Solved (failed)	Execution Time (msec) Median	Mean±SD	Max	No. Problems Solved	Execution Time (msec) Median	Mean±SD	Max	No. Problems Solved	Execution Time (msec) Median	Mean±SD	Max
DBBB$_o$	68(35)	535	16614±39141	168820	15	120	143±128	470	53	4020	21275±43285	168820
DEN$_o$	103(0)	40	88±147	1180	4	45	43±25	70	99	40	90±150	1180
G$_o$	73(30)	6230	28740±48927	195700	3	70	153±171	350	70	9365	29965±49607	195700
SR$_o$	103(0)	50	85±110	610	6	45	42±18	60	97	50	88±113	610
TR$_o$	103(0)	20	25±19	140	93	20	24±14	70	10	25	39±41	140
B2$_o$	68(35)	860	15559±32747	145580	31	340	9776±31474	145580	37	1710	20404±33426	110890
B3$_o$	12(91)	435	6961±11419	34150	7	1970	10579±13906	34150	5	410	1896±3680	8470
DBBB$_c$	14(89)	185	4536±8834	31720	14	185	4536±8834	31720	0	–	–	–
DEN$_c$	103(0)	30	55±111	1020	4	15	18±10	30	99	30	56±113	1020
G$_c$	3(100)	40	40±0	40	3	40	40±0	40	0	–	–	–
SR$_c$	103(0)	20	37±40	290	6	15	17±8	30	97	20	38±41	290
TR$_c$	94(9)	20	19±9	50	93	20	19±9	50	1	10	10±0	10
B2$_c$	25(78)	620	2243±4033	15270	25	620	2243±4033	15270	0	–	–	–
B3$_c$	3(100)	40	47±21	70	3	40	47±21	70	0	–	–	–

Figure 7.20 <u>Comparing execution times of classical and axiomatic schema translations of modal axioms in local satisfiability calculations:</u> The table shows the number of test examples (from the formulae 1-90, 105-117 in the test set; a single set of experiments was analyzed) for which execution times of the local satisfiability calculation in classical translation is, the same as, greater than, and lower than the execution time of the axiomatic schema translation. The data is also subdivided according to whether the outcome of the calculation in SPASS was Proof Found (unsatisfiable) or Completion Found (satisfiable). The mean ratios of execution times for axiomatic schema and classical translations are given. The numbers of test cases where formulae were solved in the axiomatic translation, but not in the classical translation, within the cutoff time of 200 seconds, is also reported. Table cells in which there is no data available, either because the calculation ran out of time for the all cases of the classical translation, or in all cases neither calculation produced a result, are empty.

	Execution time classical > axiomatic						Execution time classical < axiomatic					
Axiom	Complete		Total		Proof		Complete		Total		Proof	
	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio
DBBB$_o$	–	–	11	0.26	11	0.26	–	–	2	1.9	2	1.9
DEN$_o$	8	0.73	8	0.73	–	–	65	2.5	68	2.5	3	2.9
G$_o$	–	–	–	–	–	–	–	–	2	5.3	2	5.3
SR$_o$	–	–	–	–	–	–	85	2.2	91	2.3	6	2.7
TR$_o$	–	–	12	0.56	12	0.56	0	–	24	2.3	24	2.3
B^2_o	–	–	11	0.29	11	0.29	0	–	13	9.3	13	9.3
B^3_o	–	–	–	–	–	–	0	–	2	3.3	2	3.3

	Execution time classical = axiomatic			Solved in axiomatic, but not in classical		
Axiom	Complete No.	Total No.	Proof No.	Complete No.	Total No.	Proof No.

$DBBB_o$	–	1	1	53	54	1
DEN_o	26	27	1	–	–	–
G_o	–	1	1	70	70	–
SR_o	12	12	–	–	–	–
TR_o	1	58	57	9	9	–
$B^2{}_o$	–	1	1	37	43	6
$B^3{}_o$	–	1	1	8	9	4

Figure 7.21 <u>Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axioms</u>. The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 2.2 hours. A single set of data was analyzed.

Target Formula	K	T	B	D	4	5_o	alt_1	4^2	4^3	5^2	5^3	$alt_1{}^{11}$	$alt_1{}^{21}$	$alt_1{}^{12}$	$alt_1{}^{22}$
4	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
4^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
4^3	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5^3	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
D	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
T	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
alt1	C	C	C	C	C	C	P	C	C	C	C	P	P	P	P
alt1^1,1	C	C	C	C	C	C	P	C	C	C	C	P	P	P	P
alt1^1,2	C	C	C	C	C	C	P	C	C	C	C	P	P	P	P
alt1^2,1	C	C	C	C	C	C	P	C	C	C	C	P	P	P	P
alt1^2,2	C	C	C	C	C	C	P	C	C	C	C	P	P	P	P
B	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
B^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
F	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
M	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
Cxt	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
bbp,dddd(q,dnp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddnp,g	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddp,g	C	C	C	C	C	C	C	C	C	C	X	C	C	C	X
bp,ddnp,g	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,ddp	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
bq,d	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
bq,dddddddnq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bq,dddddnq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
d(bp,ddnq),ddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
d(dddddnq,ddddbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
d(ddddnq,dddbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
d(ddnq,dbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
d(dnq,dbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
d(dnq,dddbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dbp,dd(np,bq)	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
dbp,dddddnp	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
dbp,ddnp	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
dbq,d(dnq,ddp)	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
dd(bq,ddddnq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dd(dnq,dbq)	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,bbbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,ddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dddddnq,dddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dddddnq,ddddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddddnq,b(q,g)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,ddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddddnq,dddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,dbq	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
dddnq,ddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dddnq,dddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dddnq,ddddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
dddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P

ddnq,dbq	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
ddnq,ddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddnq,dddbq	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Nnd	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddp,bbbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(bp,dp)	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
n(d1d2b3bp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbbbp->bbdp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbp->bp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbdnp->nbbp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbp->dp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(ddbbp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
nd,g	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
nd	C	P	C	P	C	C	C	C	C	C	C	C	C	C	C
np,bbbp,dddddddddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,bbp,dddddddddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,ddddddddbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,dddddddddbp	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
nq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
nq,dddddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
p,bbndp,ddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
(db)^2p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^2p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
(db)^3p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^3p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
(db)^4p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^4p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
(db)^5p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^5p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
(db)^6p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^6p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
(db)^7p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^7p,q	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
(db)^8p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^8p,q	C	C	C	C	C	C	C	C	C	C	C	X	X	C	X
(db)^9p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^9p,q	C	C	C	C	C	C	C	C	C	C	C	X	X	C	X
(db)^10p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^10p,q	C	C	C	C	C	C	C	C	C	C	C	X	X	C	X
aiml02_prop3i	C	P	C	C	P	P	P	P	P	P	P	P	P	P	P
aiml02_prop3ii	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
aiml02_prop3iii	C	P	P	C	P	P	P	P	P	P	P	P	P	P	P
amai02	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
amai02b	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
demri1	C	P	C	C	P	P	C	C	P	C	P	C	P	P	X
demri2	C	C	P	C	P	P	P	P	P	P	P	P	P	P	P
demri3	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
demri5	C	C	C	C	C	C	C	C	C	C	C	C	C	C	X
demri6	C	C	C	C	C	C	C	C	X	X	X	X	X	X	X
demri7	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
demri8	C	C	C	C	P	P	P	P	P	P	P	P	P	P	P
demri9	C	C	C	C	C	C	C	C	C	C	X	C	C	X	X
CR	C	P	C	C	C	C	C	C	C	C	C	C	C	C	C
demri4	P	P	P	P	P	P	P	P	P	P	P	X	P	X	X

Figure 7.22 **<u>Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations</u>.** The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 0.2 hours. A single set of data was analyzed.

Target Formula	T4	TB	DB	D4	4_oB	5_oB	5_oT	$T4_oB$	D_o4_oB	T_c4B_c	D_cB	D_c4

4	P	P	P	P	P	P	P	P	P	P	P	P
4^2	P	P	P	P	P	P	P	P	P	P	P	P
4^3	P	P	P	P	P	P	P	P	P	P	P	P
5	P	P	P	P	P	P	P	P	P	P	P	P
5^2	P	P	P	P	P	P	P	P	P	P	P	P
5^3	P	P	P	P	P	P	P	P	P	P	P	P
D	P	P	P	P	C	C	P	P	P	P	P	P
T	P	P	P	P	C	C	P	P	P	P	P	P
alt1	C	C	C	C	C	C	C	C	C	C	C	C
alt1^1,1	C	C	C	C	C	C	C	C	C	C	C	C
alt1^1,2	C	C	C	C	C	C	C	C	C	C	C	C
alt1^2,1	C	C	C	C	C	C	C	C	C	C	C	C
alt1^2,2	C	C	C	C	C	C	C	C	C	C	C	C
B	P	P	P	P	P	P	P	P	P	P	P	P
B^2	P	P	P	P	P	P	P	P	P	P	P	P
F	C	C	C	C	C	C	C	C	C	C	C	C
M	C	C	C	C	C	C	C	C	C	C	C	C
Cxt	P	P	P	P	P	P	P	P	P	P	P	P
bbp,dddd(q,dnp)	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddnp,g	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddp,g	C	C	C	C	C	C	C	C	C	C	C	C
bp,ddnp,g	P	P	P	P	P	P	P	P	P	P	P	P
bp,ddp	C	C	C	C	C	C	C	C	C	C	C	C
bq,d	C	C	C	C	C	C	C	C	C	C	C	C
bq,dddddddnq	P	P	P	P	P	P	P	P	P	P	P	P
bq,dddddnq	P	P	P	P	P	P	P	P	P	P	P	P
d(bp,ddnq),ddbq	P	C	C	P	P	P	P	P	P	P	C	P
d(dddddnq,ddddbq)	P	C	C	P	P	P	P	P	P	P	C	P
d(ddddnq,dddbq)	P	C	C	P	P	P	P	P	P	P	C	P
d(ddnq,dbq)	P	C	C	P	P	P	P	P	P	P	C	P
d(dnq,dbq)	P	C	C	P	P	P	P	P	P	P	C	P
d(dnq,dddbq)	P	C	C	P	P	P	P	P	P	P	C	P
dbp,dd(np,bq)	P	P	P	P	P	P	P	P	P	P	P	P
dbp,dddddnp	P	P	P	P	P	P	P	P	P	P	P	P
dbp,ddnp	P	P	P	P	P	P	P	P	P	P	P	P
dbq,d(dnq,ddp)	P	P	P	P	P	P	P	P	P	P	P	P
dd(bq,ddddnq)	P	C	C	P	P	P	P	P	P	P	C	P
dd(dnq,dbq)	P	C	C	P	P	P	P	P	P	P	C	P
ddddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,bbbq	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,ddbq	P	C	C	P	P	P	P	P	P	P	C	P
dddddnq,dddbq	P	C	C	P	P	P	P	P	P	P	C	P
dddddnq,ddddbq	P	C	C	P	P	P	P	P	P	P	C	P
ddddnq,b(q,g)	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bbp	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,ddbq	P	C	C	P	P	P	P	P	P	P	C	P
ddddnq,dddbq	P	C	C	P	P	P	P	P	P	P	C	P
ddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bbq	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,dbq	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,ddbq	P	C	C	P	P	P	P	P	P	P	C	P
dddnq,dddbq	P	C	C	P	P	P	P	P	P	P	C	P
dddnq,ddddbq	P	C	C	P	P	P	P	P	P	P	C	P
dddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,dbq	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,ddbq	P	C	C	P	P	P	P	P	P	P	C	P
ddnq,dddbq	P	C	C	P	P	P	P	P	P	P	C	P
ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P
Nnd	C	C	C	C	C	C	C	C	C	C	C	C
dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dbq	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dddbq	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddp,bbbq	P	P	P	P	P	P	P	P	P	P	P	P
n(bp,dp)	P	P	P	P	C	C	P	P	P	P	P	P
n(d1d2b3bp->p)	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbbbp->bbdp)	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbp->bp)	P	P	P	P	P	P	P	P	P	P	P	P
n(dbdnp->nbbp)	P	P	P	P	P	P	P	P	P	P	P	P
n(dbp->dp)	P	P	P	P	P	P	P	P	P	P	P	P
n(ddbbp->p)	P	P	P	P	P	P	P	P	P	P	P	P
n(dp->p)	P	P	P	P	P	P	P	P	P	P	P	P
nd,g	P	P	P	P	C	C	P	P	P	P	P	P
nd	P	P	P	P	C	C	P	P	P	P	P	P
np,bbbp,ddddddddq	P	P	P	P	P	P	P	P	P	P	P	P
np,bbp,ddddddddq	P	P	P	P	P	P	P	P	P	P	P	P

np,ddddddddbp	P	P	P	P	P	P	P	P	P	P	P	P
np,dddddddddbp	C	C	C	C	C	C	C	C	C	C	C	C
nq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P
nq,dddddbq	P	P	P	P	P	P	P	P	P	P	P	P
p,bbndp,ddq	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^2p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^2p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^3p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^3p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^4p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^4p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^5p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^5p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^6p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^6p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^7p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^7p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^8p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^8p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^9p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^9p,q	C	C	C	C	C	C	C	C	C	C	C	C
(db)^10p,p	P	P	P	P	P	P	P	P	P	P	P	P
(db)^10p,q	C	C	C	C	C	C	C	C	C	C	C	C
aiml02_prop3i	P	P	C	P	P	P	P	P	P	P	C	P
aiml02_prop3ii	P	P	P	P	P	P	P	P	P	P	P	P
aiml02_prop3iii	P	P	P	P	P	P	P	P	P	P	P	P
amai02	P	P	P	P	P	P	P	P	P	P	P	P
amai02b	P	P	P	P	P	P	P	P	P	P	P	P
demri1	P	P	C	P	P	P	P	P	P	P	C	P
demri2	P	P	P	P	P	P	P	P	P	P	P	P
demri3	P	P	P	P	P	P	P	P	P	P	P	P
demri5	P	C	C	P	C	C	P	P	P	P	C	P
demri6	C	C	C	C	C	C	C	C	C	C	C	C
demri7	P	P	C	P	P	P	P	P	P	P	C	P
demri8	P	C	C	P	P	P	P	P	P	P	C	P
demri9	C	C	C	C	C	C	C	C	C	C	C	C
CR	P	P	C	C	P	P	P	P	P	P	C	C
demri4	P	P	P	P	P	P	P	P	P	P	P	P

Figure 7.23 <u>Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axioms</u>. The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 24.3 hours.

<u>Software testing</u>: It was confirmed that, where a result (Proof or Completion) was produced for both the translation of a particular formula with the axiom translated as correspondence property and axiom translated as an axiomatic schema, that there was *no difference* between these outcomes in any of the cases tested (the outcomes for T = the outcomes for T_c, $5_o = 5_c$, etc).

Target Formula	T_c	D_c	B_c	4_c	5_c	alt_{1c}	$4^2{}_c$	$4^3{}_c$	$5^2{}_c$	$5^3{}_c$	$alt_1{}^{1,1}{}_c$	$alt_1{}^{2,1}{}_c$	$alt_1{}^{1,2}{}_c$	$alt_1{}^{2,2}{}_c$
4	P	P	P	P	P	P	P	P	P	P	P	P	P	P
4^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P
4^3	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P
5^3	P	P	P	P	P	P	P	P	P	P	P	P	P	P
D	P	P	C	X	X	C	X	X	X	X	C	C	C	C
T	P	P	C	C	C	C	C	C	C	C	C	C	C	C
alt1	C	C	C	X	X	P	X	X	X	X	P	P	P	P
alt1^1,1	C	C	C	X	X	P	X	X	X	X	P	P	P	X
alt1^1,2	C	C	C	X	X	P	X	X	X	X	P	P	P	X
alt1^2,1	C	C	C	X	X	P	X	X	X	X	P	P	P	X
alt1^2,2	C	C	C	X	X	P	X	X	X	X	P	P	P	X

B	P	P	P	P	P	P	P	P	P	P	P	P	P	P
B^2	P	P	P	P	P	P	P	P	P	P	P	P	P	P
F	C	C	C	X	X	C	X	X	X	X	C	C	C	C
M	C	C	C	X	X	C	X	X	X	X	C	C	C	C
Cxt	C	C	P	P	P	P	P	P	P	X	P	P	P	P
bbp,dddd(q,dnp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddnp,g	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,dddp,g	C	C	C	X	X	X	X	X	X	X	X	X	X	X
bp,ddnp,g	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bp,ddp	C	C	C	X	X	X	X	X	X	X	X	X	X	X
bq,d	C	C	C	X	X	X	X	X	X	X	X	X	X	X
bq,ddddddddnq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
bq,dddddnq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
d(bp,ddnq),ddbq	C	C	C	X	P	P	X	X	P	X	P	P	P	X
d(ddddddnq,ddddbq)	C	C	C	X	P	P	X	X	X	X	X	X	X	X
d(ddddnq,dddbq)	C	C	C	X	P	P	X	X	X	X	X	X	X	X
d(ddnq,dbq)	C	C	C	P	P	P	P	P	P	X	P	X	X	X
d(dnq,dbq)	C	C	C	X	P	P	X	X	P	X	P	P	P	P
d(dnq,dddbq)	C	C	C	X	P	P	X	X	P	X	P	P	P	P
dbp,dd(np,bq)	C	C	P	P	P	P	P	P	P	X	P	P	P	P
dbp,dddddnp	C	C	P	X	P	P	X	X	P	X	P	P	P	P
dbp,ddnp	C	C	P	P	P	P	P	P	P	X	P	P	P	P
dbq,d(dnq,ddp)	C	C	P	X	P	P	X	X	P	X	P	P	P	P
dd(bq,ddddnq)	C	C	C	X	P	P	X	X	P	X	P	P	P	X
dd(dnq,dbq)	C	C	C	X	P	P	X	X	X	X	P	X	X	X
ddddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,bbbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddddnq,ddbq	C	C	C	X	P	P	X	X	X	X	P	P	P	X
dddddnq,dddbq	C	C	C	X	P	P	X	X	X	X	X	P	P	X
dddddnq,ddddbq	C	C	C	X	P	P	X	X	X	X	X	P	P	X
ddddnq,b(q,g)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddddnq,ddbq	C	C	C	X	P	P	X	X	P	X	P	P	P	X
ddddnq,dddbq	C	C	C	X	P	P	X	X	X	X	X	P	P	X
ddddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dddnq,dbq	C	C	P	X	P	P	X	P	P	X	P	P	P	P
dddnq,ddbq	C	C	C	X	P	P	X	X	P	X	P	P	P	X
dddnq,dddbq	C	C	C	X	P	P	X	X	X	X	P	P	P	X
dddnq,ddddbq	C	C	C	X	P	P	X	X	X	X	P	P	P	X
dddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,dbq	C	C	P	P	P	P	P	P	P	P	P	P	P	P
ddnq,ddbq	C	C	C	P	P	P	P	P	P	X	P	P	P	P
ddnq,dddbq	C	C	C	P	P	P	P	P	P	X	P	P	P	P
ddnq,dp,bbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Nnd	C	C	C	X	X	X	X	X	X	X	X	X	X	X
dnq,bq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,dddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
dnq,ddp,bbbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(bp,dp)	P	P	C	X	X	C	X	X	X	X	C	C	C	C
n(d1d2b3bp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbbbp->bbdp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbbp->bp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbdnp->nbbp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dbp->dp)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(ddbbp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
n(dp->p)	P	P	P	P	P	P	P	P	P	P	P	P	P	P
nd,g	P	P	C	X	X	C	X	X	X	X	C	C	C	C
nd	P	P	C	C	C	C	C	C	C	C	C	C	C	C
np,bbbp,ddddddddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,bbp,dddddddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,ddddddddbp	P	P	P	P	P	P	P	P	P	P	P	P	P	P
np,dddddddddbp	C	C	C	X	X	X	X	X	X	X	X	X	X	X
nq,ddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
nq,dddddbq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
p,bbndp,ddq	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^1p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^2p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^2p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^3p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^3p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^4p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^4p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X

(db)^5p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^5p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^6p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^6p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^7p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^7p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^8p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^8p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^9p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^9p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
(db)^10p,p	P	P	P	P	P	P	P	P	P	P	P	P	P	P
(db)^10p,q	C	C	C	X	X	X	X	X	X	X	X	X	X	X
aiml02_prop3i	P	C	C	P	P	P	P	P	P	X	P	P	P	P
aiml02_prop3ii	C	C	P	X	P	P	X	X	P	P	P	P	P	P
aiml02_prop3iii	P	C	P	P	P	P	P	P	P	X	P	P	P	P
amai02	P	P	P	P	P	P	P	P	P	P	P	P	P	P
amai02b	P	P	P	P	P	P	P	P	P	P	P	P	P	P
demri1	P	C	C	P	P	X	X	X	X	X	X	P	P	X
demri2	C	C	P	X	P	P	X	X	P	P	P	P	P	P
demri3	P	P	P	P	P	P	P	P	P	P	P	P	P	P
demri5	C	C	C	C	C	X	X	X	X	X	X	X	X	X
demri6	C	C	C	X	X	X	X	X	X	X	X	X	X	X
demri7	C	C	C	P	P	P	X	X	X	X	X	X	X	X
demri8	C	C	C	X	P	P	X	X	X	X	X	X	X	X
demri9	C	C	C	X	X	X	X	X	X	X	X	X	X	X
CR	P	C	C	X	X	X	X	X	X	X	X	X	X	X
demri4	P	P	P	X	X	X	X	X	X	X	X	X	X	X

Figure 7.24 <u>Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combination</u>s. The test set of formulae is seen in figure 7.4. C is Completion Found in SPASS (satisfiable), P is Proof Found in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 13.3 hours.

Software testing: Again, it was confirmed that, where a result (Proof or Completion) was produced for both the translation of a particular formula with the axiom translated as correspondence property and axiom translated as an axiomatic schema, that there was *no difference* between these outcomes in any of the cases tested (the outcomes for T4 = the outcomes for T_c4_c, etc). A single set of data was analyzed.

Target Formula	T_c4_c	T_cB_c	D_cB_c	D_c4_c	4_cB_c	5_cB_c	T_c5_c	$T_c4_cB_c$	$D_c4_cB_c$
4	P	P	P	P	P	P	P	P	P
4^2	P	P	P	P	P	P	P	P	P
4^3	P	P	P	P	P	P	P	P	P
5	P	P	P	P	P	P	P	P	P
5^2	P	P	P	P	P	P	P	P	P
5^3	P	P	P	P	P	P	P	P	P
D	P	P	P	P	X	X	P	P	P
T	P	P	P	P	C	C	P	P	P
alt1	X	C	C	X	X	X	X	X	X
alt1^1,1	X	C	C	X	X	X	X	X	X
alt1^1,2	X	C	C	X	X	X	X	X	X
alt1^2,1	X	C	C	X	X	X	X	X	X
alt1^2,2	X	C	C	X	X	X	X	X	X
B	P	P	P	P	P	P	P	P	P
B^2	P	P	P	P	P	P	P	P	P
F	X	C	C	X	X	X	X	X	X
M	X	C	C	X	X	X	X	X	X
Cxt	P	P	P	P	P	P	P	P	P
bbp,dddd(q,dnp)	P	P	P	P	P	P	P	P	P
bp,dddnp,g	P	P	P	P	P	P	P	P	P
bp,dddp,g	X	C	C	X	X	X	X	X	X
bp,ddnp,g	P	P	P	P	P	P	P	P	P
bp,ddp	X	C	C	X	X	X	X	X	X
bq,d	X	C	C	X	X	X	X	X	X
bq,ddddddnq	P	P	P	P	P	P	P	P	P
bq,dddddnq	P	P	P	P	P	P	P	P	P

d(bp,ddnq),ddbq	X	C	C	X	P	P	P	P	P
d(dddddnq,ddddbq)	X	C	C	X	X	X	P	X	X
d(ddddnq,dddbq)	X	C	C	X	X	X	P	X	X
d(ddnq,dbq)	P	C	C	P	P	P	P	P	P
d(dnq,dbq)	X	C	C	X	P	P	P	P	P
d(dnq,dddbq)	X	C	C	X	P	P	P	P	P
dbp,dd(np,bq)	P	P	P	P	P	P	P	P	P
dbp,dddddnp	X	P	P	X	P	P	P	P	P
dbp,ddnp	P	P	P	P	P	P	P	P	P
dbq,d(dnq,ddp)	X	P	P	X	P	P	P	P	P
dd(bq,ddddnq)	X	C	C	X	P	P	P	P	P
dd(dnq,dbq)	X	C	C	X	P	P	P	P	P
dddddddnq,dp,bbp	P	P	P	P	P	P	P	P	P
dddddnq,bbbq	P	P	P	P	P	P	P	P	P
dddddnq,ddbq	X	C	C	X	P	P	P	P	P
dddddnq,dddbq	X	C	C	X	X	P	P	X	X
dddddnq,ddddbq	X	C	C	X	X	X	P	X	X
ddddnq,b(q,g)	P	P	P	P	P	P	P	P	P
ddddnq,bbp	P	P	P	P	P	P	P	P	P
ddddnq,bq	P	P	P	P	P	P	P	P	P
ddddnq,ddbq	X	C	C	X	P	P	P	P	P
ddddnq,dddbq	X	C	C	X	P	P	P	P	X
ddddnq,dp,bbp	P	P	P	P	P	P	P	P	P
dddnq,bbq	P	P	P	P	P	P	P	P	P
dddnq,bq	P	P	P	P	P	P	P	P	P
dddnq,dbq	X	P	P	X	P	P	P	P	P
dddnq,ddbq	X	C	C	X	P	P	P	P	P
dddnq,dddbq	X	C	C	X	X	P	P	X	X
dddnq,ddddbq	X	C	C	X	X	X	P	P	X
dddnq,dp,bbp	P	P	P	P	P	P	P	P	P
ddnq,bq	P	P	P	P	P	P	P	P	P
ddnq,dbq	P	P	P	P	P	P	P	P	P
ddnq,ddbq	P	C	C	P	P	P	P	P	P
ddnq,dddbq	P	C	C	P	P	P	P	P	X
ddnq,dp,bbp	P	P	P	P	P	P	P	P	P
Nnd	X	C	C	X	X	X	X	X	X
dnq,bq	P	P	P	P	P	P	P	P	P
dnq,dbq	P	P	P	P	P	P	P	P	P
dnq,ddbq	P	P	P	P	P	P	P	P	P
dnq,dddbq	P	P	P	P	P	P	P	P	P
dnq,ddp,bbbq	P	P	P	P	P	P	P	P	P
n(bp,dp)	P	P	P	P	X	X	P	P	P
n(d1d2b3bp->p)	P	P	P	P	P	P	P	P	P
n(dbbbbp->bbdp)	P	P	P	P	P	P	P	P	P
n(dbbp->bp)	P	P	P	P	P	P	P	P	P
n(dbdnp->nbbp)	P	P	P	P	P	P	P	P	P
n(dbp->dp)	P	P	P	P	P	P	P	P	P
n(ddbbp->p)	P	P	P	P	P	P	P	P	P
n(dp->p)	P	P	P	P	P	P	P	P	P
nd,g	P	P	P	P	X	X	P	P	P
nd	P	P	P	P	C	C	P	P	P
np,bbbp,ddddddddq	P	P	P	P	P	P	P	P	P
np,bbp,ddddddddq	P	P	P	P	P	P	P	P	P
np,ddddddddbp	P	P	P	P	P	P	P	P	P
np,dddddddddbp	X	C	C	X	X	X	X	X	X
nq,ddbq	P	P	P	P	P	P	P	P	P
nq,dddddbq	P	P	P	P	P	P	P	P	P
p,bbndp,ddq	P	P	P	P	P	P	P	P	P
(db)^1p,p	P	P	P	P	P	P	P	P	P
(db)^1p,q	X	C	C	X	X	X	X	X	X
(db)^2p,p	P	P	P	P	P	P	P	P	P
(db)^2p,q	X	C	C	X	X	X	X	X	X
(db)^3p,p	P	P	P	P	P	P	P	P	P
(db)^3p,q	X	C	C	X	X	X	X	X	X
(db)^4p,p	P	P	P	P	P	P	P	P	P
(db)^4p,q	X	C	C	C	X	X	X	X	X
(db)^5p,p	P	P	P	P	P	P	P	P	P
(db)^5p,q	X	C	C	C	X	X	X	X	X
(db)^6p,p	P	P	P	P	P	P	P	P	P
(db)^6p,q	X	C	C	C	X	X	X	X	X
(db)^7p,p	P	P	P	P	P	P	P	P	P
(db)^7p,q	X	C	C	C	X	X	X	X	X
(db)^8p,p	P	P	P	P	P	P	P	P	P
(db)^8p,q	X	C	C	C	X	X	X	X	X
(db)^9p,p	P	P	P	P	P	P	P	P	P
(db)^9p,q	X	C	C	C	X	X	X	X	X
(db)^10p,p	P	P	P	P	P	P	P	P	P
(db)^10p,q	X	C	C	C	X	X	X	X	X
aiml02_prop3i	P	P	C	P	P	P	P	P	P

aiml02_prop3ii	P	P	P	X	P	P	P	P	P
aiml02_prop3iii	P	P	P	P	P	P	P	P	P
amai02	P	P	P	P	P	P	P	P	P
amai02b	P	P	P	P	P	P	P	P	P
demri1	P	P	C	P	P	P	P	P	P
demri2	X	P	P	X	P	P	P	P	P
demri3	P	P	P	P	P	P	P	P	P
demri5	X	C	C	X	C	C	P	P	X
demri6	X	C	C	X	X	X	X	X	X
demri7	P	P	C	X	P	P	P	P	P
demri8	X	C	C	X	P	P	P	P	P
demri9	X	C	C	X	X	X	X	X	X
CR	P	P	C	X	P	P	P	P	P
demri4	P	P	P	X	P	P	P	P	P

Figure 7.25 <u>Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations</u>. The data is taken from the results for test problems 1-119. Problems are deemed to have failed if they were not solved before running out of time at 200 seconds execution time. Execution time statistics are listed for the solved problems. The statistics for all the problems, regardless of outcome, are all listed under the heading *Total*. This set of solved problems is sub-divided by the outcome, either Completion in SPASS (satisfiable), or Proof in SPASS (unsatisfiable), in the other two sections. Where there is no data (no problems solved) the table is blank. The median, arithmetic mean ± standard deviation, and maximum values are recorded.

	Total set of problems			
		Execution time (msec)		
Axiom	**No. problems solved (failed)**	**Median**	**Mean ±SD**	**Max**
K	119(0)	20	21±20	140
T	119(0)	30	34±24	150
B	119(0)	20	22±20	170
D	119(0)	20	54±108	820
4	119(0)	20	22±20	130
5	119(0)	20	45±86	870
Alt_1	119(0)	20	54±107	800
4^2	119(0)	20	47±126	1090
4^3	118(1)	20	789±4907	47970
5^2	118(1)	40	275±1275	13270
5^3	116(3)	70	2322±7979	47990
$Alt_1^{1,1}$	114(5)	30	2118±12932	123380
$Alt_1^{2,1}$	115(4)	30	2780±15982	145230
$Alt_1^{1,2}$	116(3)	30	2595±17905	187220
$Alt_1^{2,2}$	104(15)	30	1986±11239	89390
T4	119(0)	20	24±25	190
TB	119(0)	20	23±20	150
DB	119(0)	20	51±82	600
D4	119(0)	20	50±91	680
4_oB	119(0)	20	47±96	940
5_oB	119(0)	30	59±223	2440
5_oT	119(0)	20	40±65	670
$T4_oB$	119(0)	20	44±70	600
$Do4_oB$	119(0)	50	151±526	5620
T_c4B_c	119(0)	20	27±39	330
D_cB	119(0)	20	27±27	210
D_c4	119(0)	20	25±28	200
T_c	119(0)	20	21±17	100
D_c	119(0)	20	21±17	100
B_c	119(0)	20	26±27	170
4_c	69(50)	10	232±1415	11740
5_c	90(29)	20	131±749	6920
Alt_{1c}	98(21)	20	19±12	80
4^2_c	66(53)	10	232±1621	13190
4^3_c	67(52)	20	731±4261	34260
5^2_c	77(42)	20	8417±29312	170170
5^3_c	60(59)	10	3734±25708	198810

$Alt_1^{1,1}{}_c$	91(28)	20	3443±16657	145120
$Alt_1^{2,1}{}_c$	93(26)	20	5252±19246	124790
$Alt_1^{1,2}{}_c$	93(26)	20	3302±10813	60930
$Alt_1^{2,2}{}_c$	78(41)	20	209±1185	10220
T_c4_c	74(45)	10	313±2238	19280
T_cB_c	119(0)	20	28±31	190
D_cB_c	119(0)	20	26±28	210
D_c4_c	77(42)	20	1753±6657	48170
4_cB_c	86(33)	20	884±6940	64410
5_cB_c	88(31)	20	415±1474	10040
T_c5c	95(24)	20	119±416	3590
$T_c4_cB_c$	90(29)	20	2910±19652	170350
$D_c4_cB_c$	86(33)	20	816±2547	18380

Axiom	Problem outcome = Proof				Problem outcome = Completion			
	No. problems solved	Execution time (msec)			No. problems solved	Execution time (msec)		
		Median	Mean ±SD	Max		Median	Mean ±SD	Max
K	56	10	16±10	50	63	20	25±26	140
T	65	20	25±15	60	54	40	44±28	150
B	66	10	18±11	60	53	20	28±26	170
D	61	20	26±28	190	58	30	84±146	820
4	88	20	19±10	60	31	20	33±34	130
5	88	20	33±34	280	31	30	78±155	870
Alt_1	92	20	32±35	230	27	30	128±201	800
4^2	87	20	22±18	140	32	20	115±230	1090
4^3	88	20	45±170	1610	30	20	2970±9510	47970
5^2	87	40	80±126	790	31	80	823±2424	13270
5^3	88	60	728±2841	21780	28	720	7332±14520	47990
$Alt_1^{1,1}$	91	30	45±71	570	23	30	10323±27763	123380
$Alt_1^{2,1}$	93	30	898±8011	77300	22	55	10733±31992	145230
$Alt_1^{1,2}$	92	30	327±1639	13800	24	100	11291±38638	187220
$Alt_1^{2,2}$	91	30	2125±11979	89390	13	20	1018±2627	8910
T4	95	20	19±10	60	24	20	44±47	190
TB	75	10	18±11	60	44	20	32±27	150
DB	71	20	26±21	110	48	40	88±117	600
D4	94	20	31±31	230	25	30	122±174	680
4_oB	89	20	33±35	220	30	20	89±177	940
5_oB	89	30	37±31	170	30	30	127±439	2440
5_oT	95	20	30±23	130	24	30	79±134	670
$T4_oB$	95	20	31±28	160	24	35	95±137	600
$Do4_oB$	95	50	84±105	570	24	70	418±1131	5620
T_c4B_c	95	20	20±13	80	24	20	57±76	330
D_cB	71	10	17±11	60	48	25	40±37	210
D_c4	94	20	20±13	90	25	20	47±51	200
T_c	65	10	16±11	80	54	20	27±21	100
D_c	61	10	16±12	80	58	20	26±20	100
B_c	66	10	16±9	50	53	20	37±35	170
4_c	66	15	225±1442	11740	3	10	390±658	1150
5_c	87	20	56±198	1390	3	10	2313±3989	6920
Alt_{1c}	91	20	19±12	80	7	10	13±5	20
$4^2{}_c$	64	15	239±1646	13190	2	10	10±0	10
$4^3{}_c$	65	20	753±4325	34260	2	10	10±0	10
$5^2{}_c$	75	20	8642±29673	170170	2	10	10±0	10
$5^3{}_c$	58	15	3862±26145	198810	2	10	10±0	10
$Alt_1^{1,1}{}_c$	84	20	3729±17314	145120	7	10	14±8	30
$Alt_1^{2,1}{}_c$	86	20	5678±19962	124790	7	10	23±22	60
$Alt_1^{1,2}{}_c$	86	20	3570±11207	60930	7	10	17±13	40
$Alt_1^{2,2}{}_c$	71	20	228±1241	10220	7	10	19±16	50
T_c4_c	74	10	313±2238	19280	0	–	–	0
T_cB_c	75	10	19±23	190	44	30	43±37	150
D_cB_c	71	10	17±11	60	48	25	40±37	210
D_c4_c	70	10	1918±6965	48170	7	90	100±57	190
4_cB_c	83	20	906±7065	64410	3	10	270±450	790
5_cB_c	85	20	419±1498	10040	3	10	290±485	850
T_c5c	95	20	119±416	3590	0	–	–	–
$T_c4_cB_c$	90	20	2910±19652	170350	0	–	–	–
$D_c4_cB_c$	86	20	816±2547	18380	0	–	–	–