

22*	bp, ddnq, g	$\wedge(\wedge(\Box p, \Diamond \neg p), \wedge(\Diamond \Diamond \Diamond \Diamond q_1, \Diamond \Diamond \Diamond \Diamond q_2))$
23	bp, ddp	$\wedge(\Box p, \Diamond p)$
24	bq, d	$\wedge(\Box p, \Diamond \top)$
25*	bq, ddddddng	$\wedge(\Box p, \Diamond \Diamond \Diamond \Diamond \neg p)$
26*	bq, ddddddng	$\wedge(\Box p, \Diamond \Diamond \Diamond \neg p)$
27	d(bp, ddnq), ddbq	$\wedge(\Diamond(\wedge(\Box p, \Diamond \neg q)), \Diamond \Box q)$
28	d(dddndq, ddddbq)	$\Diamond(\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q))$
29	d(dddndq, ddbq)	$\Diamond(\wedge(\Diamond \Diamond \neg q, \Diamond \Box q))$
30	d(dndq, dbq)	$\Diamond(\wedge(\Diamond \neg q, \Box q))$
31	d(dnq, dbq)	$\Diamond(\wedge(\neg q, \Box q))$
32	d(dnq, ddbq)	$\Diamond(\wedge(\neg q, \Diamond \Box q))$
33	dbp, dd(np, bq)	$\wedge(\Diamond p, \Diamond(\wedge(\neg p, \Box q)))$
34	dbp, ddddnq	$\wedge(\Diamond p, \Diamond \Diamond \neg p)$
35	dbp, ddnq	$\wedge(\Diamond p, \Diamond \neg p)$
36	dbq, d(dnq, ddp)	$\wedge(\Diamond(\wedge(\Diamond \neg q, \Diamond p)), \Diamond \Box q)$
37	dd(bq, ddddnq)	$\Diamond \Diamond(\wedge(\Box p, \Diamond \Diamond \neg p))$
38	dd(dnq, dbq)	$\Diamond \Diamond(\wedge(\Diamond \neg q, \Box q))$
39*	dddndnq, dp, bbp	$\wedge(\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond p), \Box \Box q)$
40*	dddndnq, bbbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box \Box q)$
41	dddndnq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Box q)$
42	dddndnq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q)$
43	dddndnq, ddddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q)$
44*	dddndq, b(q, g)	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box(\wedge(p, q)))$
45*	dddndq, bbp	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box q)$
46*	dddndq, bq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box q)$
47	dddndq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Box q)$
48	dddndq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q)$
49*	dddndq, dp, bbp	$\wedge(\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond p), \Box \Box q)$
50*	dddndq, bbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box \Box q)$
51*	dddndq, bq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Box q)$
52	dddndq, dbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Box q)$
53	dddndq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Box q)$
54	dddndq, ddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q)$
55	dddndq, ddddbq	$\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond \Diamond \Box q)$
56*	dddndq, dp, bbp	$\wedge(\wedge(\Diamond \Diamond \Diamond \neg q, \Diamond p), \Box \Box q)$
57*	dndq, bq	$\wedge(\Diamond \neg q, \Box q)$
58	dndq, dbq	$\wedge(\Diamond \neg q, \Diamond \Box q)$
59	dndq, ddbq	$\wedge(\Diamond \neg q, \Diamond \Box q)$
60	dndq, ddbq	$\wedge(\Diamond \neg q, \Diamond \Diamond \Box q)$
61	dndq, dp, bbp	$\wedge(\wedge(\Diamond \neg q, \Diamond p), \Box \Box q)$
62	Nnd	$\neg(\neg \Diamond \top)$
63	dnq, bq	$\wedge(\Diamond \neg q, \Box q)$
64*	dnq, dbq	$\wedge(\Diamond \neg q, \Diamond \Box q)$
65	dnq, ddbq	$\wedge(\Diamond \neg q, \Diamond \Box q)$
66	dnq, ddbq	$\wedge(\Diamond \neg q, \Diamond \Diamond \Box q)$
67	dnq, ddp, bbbq	$\wedge(\wedge(\Diamond \neg q, \Diamond p), \Box \Box q)$
68	n(bp, dp)	$\neg(\rightarrow(\Box p, p))$
69	n(d1d2b3bp->p)	$\neg(\rightarrow(\Diamond(\wedge(p_1, \Diamond(\wedge(p_2, \Box(\wedge(p_3, \Box p)))))), p))$
70	n(dbbbbp->bbdp)	$\neg(\rightarrow(\Diamond \Box \Box \Box p, \Box \Diamond p))$
71	n(dbbp->bp)	$\neg(\rightarrow(\Diamond \Box p, p))$
72	n(dbdnp->nbbp)	$\neg(\rightarrow(\Diamond \Diamond \neg p, \neg \Box p))$
73	n(dbp->dp)	$\neg(\rightarrow(\Diamond p, p))$
74	n(dbbp->p)	$\neg(\rightarrow(\Diamond \Box p, p))$
75	n(dp->p)	$\neg(\rightarrow(\Diamond p, p))$
76	nd, g	$\neg(\Diamond(\vee(p, \neg p)))$
77	nd	$\neg \Diamond \top$
78	np, bbbp, ddddddng	$\wedge(\wedge(\neg p, \Box \Box p), \Diamond \Diamond \Diamond \Diamond \Diamond q)$
79	np, bbp, ddddddng	$\wedge(\wedge(\neg p, \Box p), \Diamond \Diamond \Diamond \Diamond q)$
80	np, ddddddng	$\wedge(\neg p, \Diamond \Diamond \Diamond \Diamond \Box p)$
81	np, ddddddng	$\wedge(\neg p, \Diamond \Diamond \Diamond \Diamond \Box q)$
82	ng, dbq	$\wedge(\neg q, \Diamond \Box q)$

83	ng, ddddbq	$\wedge(\neg p, \Diamond \Diamond \Diamond \Box p)$
84	p, bbdnp, ddq	$\wedge(p, \wedge(\Box(\neg(\neg p))), \Diamond(\Diamond q))$
85	(db) ^1p, p	$\neg(\rightarrow(\Diamond p, p))$
86	(db) ^1p, q	$\neg(\rightarrow(\Diamond p, q))$
87	(db) ^2p, p	$\neg(\rightarrow(\Diamond \Box p, p))$
88	(db) ^2p, q	$\neg(\rightarrow(\Diamond \Box p, q))$
89	(db) ^3p, p	$\neg(\rightarrow(\Diamond \Diamond \Box p, p))$
90	(db) ^3p, q	$\neg(\rightarrow(\Diamond \Diamond \Box p, q))$
91	(db) ^4p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box p, p))$
92	(db) ^4p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box p, q))$
93	(db) ^5p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box p, p))$
94	(db) ^5p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box p, q))$
95	(db) ^6p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box p, p))$
96	(db) ^6p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box p, q))$
97	(db) ^7p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box p, p))$
98	(db) ^7p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box p, q))$
99	(db) ^8p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box p, p))$
100	(db) ^8p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box p, q))$
101	(db) ^9p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box \Box p, p))$
102	(db) ^9p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box \Box p, q))$
103	(db) ^10p, p	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box \Box \Box p, p))$
104	(db) ^10p, q	$\neg(\rightarrow(\Diamond \Diamond \Box \Box \Box \Box \Box \Box \Box \Box p, q))$
105	aim102_prop3i	$\neg(\rightarrow(\Box(\rightarrow(\Box p, \Box q)), \Box(\rightarrow(\Box p, q))))$
106	aim102_prop3ii	$\neg(\rightarrow(\Box(\rightarrow(\Box p, q)), \Box(\rightarrow(\Box p, \Box q))))$
107*	aim102_prop3iii	$\neg(\rightarrow(\Diamond(\Box(\rightarrow(\Box p, q))), \Box(\rightarrow(\Box p, \Diamond q))))$
108*	amai02	$\neg(\rightarrow(\Box(\rightarrow(\Box p, q)), \Box(\rightarrow(\Box p, \Box(\rightarrow(\Box p, q))))))$
109*	amai02b	$\neg(\rightarrow(\neg(\Box(\rightarrow(\Box p, q))), \Box(\rightarrow(\Box p, \neg(\Box(\rightarrow(\Box p, q)))))))$
110*	demri1	$\neg(\leftrightarrow(\Box(\vee(\Box p, \Box q)), \vee(\Box p, \Box q)))$
111*	demri2	$\neg(\rightarrow(\Box(\rightarrow(\Box(\leftrightarrow(p, q)), t)), \Box(\leftrightarrow(p, q)), \Box t))$
112	demri3	$\neg(\leftrightarrow(\Box p, \wedge(\Box(\rightarrow(q, p))), \Box(\rightarrow(\neg q, p))))$
113	demri5	$\neg(\Diamond(\Box(\leftrightarrow(\Box(\vee(p, \Box q)), \vee(\Box p, \Box q))))$
114	demri6	$\neg(\Diamond(\Box(\leftrightarrow(\rightarrow(p, q), \vee(\neg q, \vee(\wedge(\vee(\neg p, \vee(\wedge(q, \Diamond(\wedge(p, \neg q))))), \neg(\wedge(p, q))))))), \neg(\Diamond(\wedge(p, q))))), \Diamond(\wedge(q, \neg(\vee(\neg p, \vee(\wedge(q, \Diamond(\wedge(p, \neg q))))), \neg(\wedge(p, q))))))$
115	demri7	$\neg(\Box(\rightarrow(\Box(\rightarrow(\Box p, \Box(\rightarrow(\Box q, \Box t))), \Box(\rightarrow(\Box q, \Box(\rightarrow(\Box p, \Box t))))))$
116	demri8	$\neg(\Box(\rightarrow(\Box(\rightarrow(\Box p, \Box(\rightarrow(\Box q, \Box t))), \Box(\rightarrow(\Box q, \Box(\rightarrow(\Box p, \Box t))))))$
117	demri9	$\neg(\Box(\rightarrow(\Box(\rightarrow(\Box p, \Box(\rightarrow(\Box q, \Box t))), \Box(\rightarrow(\Box(\rightarrow(\Box p, \Box q)), \Box t))))$
118*	CR	$\neg(\rightarrow(\Diamond(x, \Box(a, p)), \Box(a, \Diamond(x, p))))$
119	demri4	$\neg(\rightarrow(\wedge(\Box(x_c, \rightarrow(\neg p_c, \Box(x_b, \neg p_c))), \wedge(\Box(x_c, \Box(x_b, \Box(x_a, \vee(p_a, \vee(p_b, p_c)))))), \wedge(\Box(x_c, \Box(x_b, \rightarrow(\neg p_b, \Box(x_a, \neg p_b))))), \wedge(\Box(x_c, \Box(x_b, \rightarrow(\neg p_c, \Box(x_a, \neg p_c))))), \wedge(\Box(x_c, \neg(\Box(x_b, p_b))), \Box(x_c, \Box(x_b, \neg(\Box(x_a, p_a))))))$
120	bbf	$\Box \Box \perp$
121	bbt	$\Box \Box \top$
122	bdf	$\Box \Diamond \perp$
123	bdT	$\Box \Diamond \top$
124	bf	$\Box \perp$
125	bt	$\Box \top$
126	dbf	$\Diamond \Box \perp$
127	dbT	$\Diamond \Box \top$
128	ddf	$\Diamond \Diamond \perp$
129	ddT	$\Diamond \Diamond \top$
130	df	$\Diamond \perp$
131	dt	$\Diamond \top$
132	fa	\perp or $\neg \top$
133	tr	\top or $\neg \perp$

Figure 7.5: Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed single axioms. The test set of formulae is listed in figure 7.4. C is Completion in SPASS (satisfiable), P is Proof in SPASS (unsatisfiable), and X is ran out of time at 200 seconds execution time. The total execution time for the data in this table is 22.2 hours. The data was extracted from saved output files using a c-shell script, and processed in Microsoft Excel.

2	4^2	P	C	C	C	P	X	X	X	X	X
3	4^3	P	C	C	C	P	X	X	X	X	X
4	5	X	C	C	C	X	P	P	P	P	P
5	5^2	X	C	C	C	X	P	P	P	P	P
6	5^3	X	C	C	C	X	P	P	P	P	P
7	D	P	P	P	P	X	X	P	P	P	P
8	T	P	P	C	X	X	X	P	P	P	P
9	alt1	X	C	C	C	X	X	X	X	X	X
10	alt1^1,1	X	C	C	C	X	X	X	X	X	X
11	alt1^1,2	X	C	C	C	X	X	X	X	X	X
12	alt1^2,1	X	C	C	C	X	X	X	X	X	X
13	alt1^2,2	X	C	C	C	X	X	X	X	X	X
14	B	X	P	P	P	X	P	P	P	P	P
15	B^2	X	C	C	C	X	P	P	P	P	P
16	F	X	C	C	C	X	X	X	X	X	X
17	M	X	C	C	C	X	X	X	X	X	X
18	Cxt	X	C	C	C	X	P	P	P	P	P
19	bbp, dddd(q, dnp)	P	C	C	C	P	X	X	X	X	X
20	bp, dddnp, g	X	C	C	C	P	X	X	X	X	X
21	bp, dddp, g	X	C	C	C	X	X	X	X	X	X
22	bp, ddnq, g	X	C	C	C	X	X	X	X	X	X
23	bp, ddp	X	C	C	C	X	X	X	X	X	X
24	bq, d	X	C	C	C	X	X	X	X	X	X
25	bq, ddddddnq	X	C	C	C	P	X	X	X	X	X
26	bq, ddddndq	P	C	C	C	P	X	X	X	X	X
27	d(bp, ddnq), ddbq	X	C	C	C	X	P	P	P	P	X
28	d(ddddndq, ddddbq)	X	C	C	C	X	P	P	P	X	X
29	d(ddddndq, ddddbq)	X	C	C	C	X	P	P	P	P	X
30	d(ddndq, dbq)	X	C	C	C	X	P	P	P	P	P
31	d(dnq, dbq)	X	C	C	C	X	P	P	P	P	P
32	d(dnq, ddbq)	X	C	C	C	X	P	P	P	P	P
33	dbp, dd(np, bq)	X	C	C	C	X	P	P	P	P	P
34	dbp, ddddnq	X	C	C	C	X	P	P	P	P	P
35	dbp, ddnq	X	C	C	C	X	P	P	P	P	P
36	dbq, d(dnq, ddp)	X	C	C	C	X	P	P	P	P	P
37	dd(bq, ddddnq)	P	C	C	C	P	P	P	P	X	X
38	dd(dnq, dbq)	X	C	C	C	X	P	P	P	P	P
39	dddddnq, dp, bbp	X	C	C	C	P	X	X	X	X	X
40	dddddnq, bbbq	P	C	C	C	P	X	X	P	X	X
41	dddddnq, ddbq	X	C	C	C	X	P	P	P	P	P
42	dddddnq, ddbq	X	C	C	C	X	P	P	P	X	X
43	dddddnq, ddddbq	X	C	C	C	X	P	P	P	X	X
44	dddddnq, b(q, g)	P	C	C	C	P	X	X	X	X	X
45	dddddnq, bbp	P	C	C	C	P	X	X	X	X	X
46	dddddnq, bq	P	C	C	C	P	X	X	X	X	X
47	dddddnq, ddbq	X	C	C	C	X	P	P	P	P	P
48	dddddnq, ddbq	X	C	C	C	X	P	P	P	P	X
49	dddddnq, dp, bbp	P	C	C	C	P	X	X	X	X	X
50	dddnq, bbq	P	C	C	C	P	X	P	P	X	X
51	dddnq, bq	P	C	C	C	P	X	X	X	X	X
52	dddnq, dbq	X	C	C	C	X	P	P	P	P	X
53	dddnq, ddbq	X	C	C	C	X	P	P	P	P	P
54	dddnq, ddbq	X	C	C	C	X	P	P	P	P	X
55	dddnq, ddddbq	X	C	C	C	X	P	P	P	X	X
56	dddnq, dp, bbp	P	C	C	C	P	X	P	P	X	X
57	ddnq, bq	P	C	C	C	P	P	P	P	P	X
58	ddnq, dbq	X	C	C	C	X	P	P	P	P	P
59	ddnq, ddbq	X	C	C	C	X	P	P	P	P	P
60	ddnq, ddbq	X	C	C	C	X	P	P	P	P	X
61	ddnq, dp, bbp	P	P	P	P	P	P	P	P	P	P
62	Nnd	X	C	C	C	X	X	X	X	X	X
63	dnq, bq	P	P	P	P	P	P	P	P	P	P
64	dnq, dbq	X	C	C	C	X	P	P	P	P	P
65	dnq, ddbq	X	C	C	C	X	P	P	P	P	P
66	dnq, ddbq	X	C	C	C	X	P	P	P	P	P
67	dnq, ddp, bbbq	P	P	P	X	P	P	P	P	P	P
68	n(bp, dp)	P	P	P	P	X	X	P	P	P	P
69	n(d1d2b3bp->p)	X	P	P	X	P	P	P	P	P	P
70	n(dbbbp->bbdp)	X	P	P	X	P	P	P	P	P	P
71	n(dbbp->bbp)	X	P	P	X	P	P	P	P	P	P
72	n(dbdnp->nbbp)	P	P	C	P	P	P	P	P	P	P
73	n(dbp->dp)	P	P	C	P	P	P	P	P	P	P
74	n(dbbp->p)	X	P	P	X	P	P	P	P	P	P

75	n(dp->p)	X	C	C	C	X	X	X	X	X	X
76	nd, g	P	P	P	P	P	X	X	P	P	P
77	nd	P	P	P	P	P	X	X	P	P	P
78	np, bbbp, dddddddq	P	P	C	X	P	P	P	P	P	P
79	np, bbbp, dddddddq	P	P	C	X	P	P	P	P	P	P
80	np, dddddddbp	X	C	C	X	P	P	P	P	P	P
81	np, dddddddbp	X	C	C	X	X	X	X	X	X	X
82	nq, ddbq	X	C	C	X	P	P	P	P	P	P
83	nq, ddddbq	X	C	C	X	P	P	P	P	P	P
84	p, bbndp, ddq	P	P	C	X	P	P	P	P	P	P
85	(db)^1p, p	X	P	P	X	P	P	P	P	P	P
86	(db)^1p, q	X	C	C	X	X	X	X	X	X	X
87	(db)^2p, p	X	P	P	X	P	P	P	P	P	P
88	(db)^2p, q	X	C	C	X	X	X	X	X	X	X
89	(db)^3p, p	X	P	P	X	P	P	P	P	P	P
90	(db)^3p, q	X	C	C	X	X	X	X	X	X	X
91	(db)^4p, p	X	P	P	C	P	P	P	P	P	P
92	(db)^4p, q	X	C	C	C	X	X	X	C	X	X
93	(db)^5p, p	X	P	P	C	P	P	P	P	P	P
94	(db)^5p, q	X	C	C	C	X	X	X	C	X	X
95	(db)^6p, p	X	P	P	C	P	P	P	P	P	P
96	(db)^6p, q	X	C	C	C	X	X	X	C	X	X
97	(db)^7p, p	X	P	P	C	P	P	P	P	P	P
98	(db)^7p, q	X	C	C	C	X	X	X	C	X	X
99	(db)^8p, p	X	P	P	C	P	P	P	P	P	P
100	(db)^8p, q	X	C	C	C	X	X	X	C	X	X
101	(db)^9p, p	X	P	P	C	P	P	P	P	P	P
102	(db)^9p, q	X	C	C	C	X	X	X	C	X	X
103	(db)^10p, p	X	P	P	C	P	P	P	P	P	P
104	(db)^10p, q	X	C	C	C	X	X	X	C	X	X
105	aiml02_prop3i	P	C	C	X	P	P	P	P	P	P
106	aiml02_prop3iii	X	C	C	X	P	P	P	P	P	X
107	aiml02_prop3iiii	X	C	C	X	P	P	P	P	P	P
108	ama02	P	C	C	P	P	P	P	P	P	X
109	ama02b	X	C	C	X	P	P	P	P	P	P
110	demr11	P	C	C	X	X	X	X	X	X	X
111	demr12	P	C	C	X	P	P	P	X	X	X
112	demr13	P	P	P	P	P	P	P	P	P	X
113	demr15	P	C	C	X	X	X	X	X	X	X
114	demr16	X	C	C	X	X	X	X	X	X	X
115	demr17	P	C	C	X	P	P	P	P	P	P
116	demr18	X	C	C	X	P	P	P	P	P	P
117	demr19	X	C	C	X	X	X	X	X	X	X
118	CR*	X	C	C	X	X	X	X	X	X	X
119	demr14*	X	P	P	X	X	X	X	X	X	X
120	bbf	P	P	P	P	C	C	P	P	P	P
121	bbt	C	C	C	C	C	C	C	C	C	C
122	bdf	P	P	P	P	X	X	P	P	P	P
123	bdq	X	C	C	X	X	X	X	C	X	X
124	bf	P	P	P	P	X	X	P	P	P	P
125	bt	C	C	C	C	C	C	C	C	C	C
126	dbf	P	P	P	P	P	P	P	P	P	P
127	dbt	X	C	C	C	X	X	X	C	X	X
128	ddf	P	P	P	P	P	P	P	P	P	P
129	ddt	X	C	C	C	X	X	X	C	X	X
130	df	P	P	P	P	P	P	P	P	P	P
131	dt	X	C	C	C	X	X	X	C	X	X
132	fa	P	P	P	P	P	P	P	P	P	P
133	tr	C	C	C	C	C	C	C	C	C	C

*Axiom applied to all the modality indices in the target formula.

Figure 7.9. Local satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations: Execution times. The data is taken from the results for test problems 1-119 (see figure 7.4 for details), and is an analysis of the same experiments reported in figures 7.5 to 7.8. Problems are deemed to have failed if they were not solved before running out of time at 200 seconds execution time. Execution time statistics are listed for the solved problems. The statistics for all the problems, regardless of outcome, are all listed under the heading *Total*. This set of solved problems is sub-divided by the outcome, either Completion in SPASS (satisfiable), or Proof in SPASS (unsatisfiable), in the other two sections. Where there is

no data (no problems solved) the table is blank. The median, arithmetic mean \pm standard deviation, and maximum values are recorded.

Axiom	Total set of problems				Problem outcome = Proof				Problem outcome = Completion			
	No. Problems Solved (failed)	Execution Time in msec			No. Problems Solved	Execution Time in msec			No. Problems Solved	Execution Time in msec		
		Median	Mean \pm SD	Max		Median	Mean \pm SD	Max		Median	Mean \pm SD	Max
K	119(0)	20	27 \pm 27	160	4	15	28 \pm 29	70	115	20	27 \pm 27	160
T	119(0)	40	57 \pm 34	200	15	40	39 \pm 15	80	104	50	59 \pm 35	200
B	119(0)	20	91 \pm 318	2960	21	20	52 \pm 100	480	98	20	99 \pm 347	2960
D	119(0)	30	97 \pm 178	1050	8	10	54 \pm 112	330	111	30	100 \pm 182	1050
4	119(0)	20	288 \pm 1140	8230	26	20	24 \pm 18	90	93	20	361 \pm 1281	8230
5	119(0)	60	510 \pm 1802	16250	53	50	91 \pm 121	630	66	110	847 \pm 2372	16250
alt ₁	119(0)	30	117 \pm 222	1290	19	30	48 \pm 87	400	100	40	131 \pm 237	1290
4 ²	119(0)	40	5194 \pm 24785	185280	13	20	44 \pm 69	270	106	45	5825 \pm 26204	185280
4 ³	94(25)	155	11972 \pm 33607	158360	10	35	772 \pm 2325	7390	84	310	13306 \pm 35327	158360
5 ²	74(45)	555	10568 \pm 27215	193690	27	180	1022 \pm 3449	18120	47	2910	16051 \pm 32931	193690
5 ³	23(96)	1140	7494 \pm 21355	102720	17	1770	9373 \pm 24644	102720	6	590	2170 \pm 4147	10560
alt ₁ ^{1,1}	90(29)	180	11428 \pm 27917	139520	12	45	62 \pm 53	180	78	670	13177 \pm 29624	139520
alt ₁ ^{2,1}	85(34)	280	8563 \pm 21385	117540	30	115	2665 \pm 12537	68900	55	560	11781 \pm 24436	117540
alt ₁ ^{1,2}	67(52)	400	6194 \pm 20623	142010	24	380	5158 \pm 17993	85240	43	400	6773 \pm 22138	142010
alt ₁ ^{2,2}	27(92)	4050	8348 \pm 13845	52120	8	1830	10204 \pm 18318	52120	19	6390	7567 \pm 12009	49740
T4	119(0)	20	70 \pm 170	1110	42	20	26 \pm 19	100	77	20	94 \pm 208	1110
TB	119(0)	20	67 \pm 206	1910	30	20	29 \pm 26	100	89	20	79 \pm 236	1910
DB	119(0)	40	877 \pm 5255	52470	25	40	103 \pm 150	550	94	40	1084 \pm 5902	52470
D4	119(0)	40	454 \pm 1663	11790	33	30	51 \pm 78	420	86	50	609 \pm 1936	11790
4 _B	119(0)	40	379 \pm 1449	11190	87	30	143 \pm 823	7710	32	60	1021 \pm 2352	11190
5 _B	118(1)	50	1229 \pm 9201	97430	86	50	93 \pm 137	930	32	95	4282 \pm 17501	97430
5 _T	119(0)	50	310 \pm 1495	14360	93	40	149 \pm 759	7350	26	95	885 \pm 2827	14360
T4 _B	119(0)	40	408 \pm 1832	16570	93	30	71 \pm 185	1710	26	85	1614 \pm 3711	16570
D ₄ _B	116(3)	130	3729 \pm 16585	116210	93	110	1474 \pm 12032	116210	23	270	12845 \pm 26907	109570
T ₄ _B	119(0)	30	156 \pm 490	4260	93	20	58 \pm 105	730	26	55	507 \pm 963	4260
D _B	119(0)	30	102 \pm 330	3080	25	20	35 \pm 40	200	94	30	119 \pm 370	3080
D ₄	119(0)	30	103 \pm 246	1670	33	20	32 \pm 36	170	86	30	131 \pm 285	1670

Axiom	Total set of problems				Problem outcome = Proof				Problem outcome = Completion			
	No. Problems Solved (failed)	Execution Time in msec			No. Problems Solved	Execution Time in msec			No. Problems Solved	Execution Time in msec		
		Median	Mean \pm SD	Max		Median	Mean \pm SD	Max		Median	Mean \pm SD	Max
T _c	119(0)	20	30 \pm 28	200	15	10	28 \pm 48	200	104	20	31 \pm 24	120
B _c	119(0)	30	107 \pm 304	2720	21	30	46 \pm 43	160	98	30	121 \pm 334	2720
D _c	119(0)	20	28 \pm 23	120	8	10	24 \pm 35	110	111	20	28 \pm 23	120
4 _c	25(94)	70	464 \pm 1221	6110	25	70	464 \pm 1221	6110	0	-	-	-
5 _c	48(71)	115	424 \pm 820	3690	48	115	424 \pm 820	3690	0	-	-	-
alt _{1c}	18(101)	40	55 \pm 45	180	18	40	55 \pm 45	180	0	-	-	-
4 _c ²	12(107)	595	19965 \pm 40528	120080	12	595	19965 \pm 40528	120080	0	-	-	-
4 _c ³	6(113)	16700	23457 \pm 29637	73950	6	16700	23457 \pm 29637	73950	0	-	-	-
5 _c ²	15(104)	1140	23082 \pm 36088	101480	15	1140	23082 \pm 36088	101480	0	-	-	-
5 _c ³	5(114)	90	78222 \pm 107069	197470	5	90	78222 \pm 107069	197470	0	-	-	-
alt _{1c} ^{1,1}	9(110)	8980	31962 \pm 52804	159310	9	8980	31962 \pm 52804	159310	0	-	-	-
alt _{1c} ^{2,1}	7(112)	1050	46879 \pm 79159	169940	7	1050	46879 \pm 79159	169940	0	-	-	-
alt _{1c} ^{1,2}	4(115)	70	29043 \pm 57985	116020	4	70	29043 \pm 57985	116020	0	-	-	-
alt _{1c} ^{2,2}	3(116)	110	57 \pm 81	150	3	110	57 \pm 81	150	0	-	-	-
T _c ₄	36(83)	110	10392 \pm 33343	180490	36	110	10392 \pm 33343	180490	0	-	-	-
T _c _B	119(0)	30	83 \pm 174	1200	30	20	57 \pm 122	670	89	30	91 \pm 188	1200

D _c B _c	119(0)	30	100 \pm 319	2920	25	20	35 \pm 38	190	94	30	118 \pm 357	2920
D _c 4 _c	42(77)	665	5545 \pm 14455	66820	28	1135	8042 \pm 17256	66820	14	360	551 \pm 517	1560
4 _c B _c	69(50)	120	466 \pm 1062	6160	69	120	466 \pm 1062	6160	0	-	-	-
5 _c B _c	71(48)	80	2363 \pm 12054	89040	71	80	2363 \pm 12054	89040	0	-	-	-
T _c 5 _c	77(42)	140	481 \pm 1677	14280	77	140	481 \pm 1677	14280	0	-	-	-
T _c 4 _c B _c	68(51)	90	1850 \pm 12338	101890	68	90	1850 \pm 12338	101890	0	-	-	-
D _c 4 _c B _c	57(62)	320	3322 \pm 11751	80490	57	320	3322 \pm 11751	80490	0	-	-	-

Figure 7.10 Axiomatic translation of the test formulae: Summary of counter-examples: The formulae that show discrimination between potentially complete and non-complete formulations of the axiom combination are shown (that is, the complete formulation yields the result Proof or Completion, and the incompleteness formulation yields the result Completion or Proof, respectively). Formulations of axiom combinations showing non-complete examples are in brackets. Refer to figure 7.4 for the actual formula. Formulae 1-117 were screened. The total execution time for the data in this table is 68.2 hours.

Discrimination between formulation of modal axioms	Discriminating Formulae (counter-examples)
[K5] vs K5 _o	bbp, dddd(q, dnp); ddddndq, dp, bbp; ddddndq, bbq; ddddndq, bbp; ddddndq, dp, bbp; ddddndq, bbq; ddddndq, dp, bbp
[5 ²] vs 5 ² _o	dddndq, dp, bbp; ddddndq, bbq; ddddndq, dp, bbp
[5 ³] vs 5 ³ _o	None
[4B / B ₄] vs 4 _B	5; aiml02_prop3iii; amai02b; dng, dbq
4 _B 5 _o / 4 _o 5 _B / 5 _o 4 _B / 5 _o 4 _o B / B ₄ 5 _o / B ₅ 4 _o	None
B _o T / T _B / BT	None
D45 / D ₄ 5 _o / D _o 5 ₄ / 4 _o D ₅ / 4 _o 5 _o D / 5 _o D ₄ / 5 _o 4 _o D	None
[B ₅] vs 5 _B	4; 4 ² ; 4 ³ ; amai02; bp, dddnp, g; bq, ddddndq; bq, ddddndq; ddddndq, b(q, g); ddddndq, bq; ddddndq, bq; ddddndq, bq; demri1; demri2
[T4B / TB ₄ / B ₄ T / B _o T ₄] vs T ₄ B / 4 _B T / 4 _o TB	5; aiml02_prop3iii; amai02b; dng, dbq
DT / D _o T / TD	None
[T5 _o] vs 5 _o T	4; 4 ² ; 4 ³ ; amai02; bp, dddnp, g; bq, ddddndq; bq, ddddndq; ddddndq, b(q, g); ddddndq, bq; ddddndq, bq; ddddndq, bq; demri1; demri2
[TB ₅] vs B ₅ T / T5 _B / 5 _B T	4; 4 ² ; 4 ³ ; amai02; bp, dddnp, g; bq, ddddndq; bq, ddddndq; ddddndq, b(q, g); ddddndq, bq; ddddndq, bq; demri1; demri2
[B ₅ D / B _o D ₅ / D _o B ₅] vs D ₅ B / 5 _D B / 5 _o B _D	4; 4 ² ; 4 ³ ; amai02; bq, ddddndq; bq, ddddndq; ddddndq, b(q, g); ddddndq, bq; ddddndq, bq; demri1; demri2; T
[D4B / B _o D ₄ / B ₄ D] vs D _o B ₄ / D _o B / 4 _o B / 4 _B D	5; aiml02_prop3iii; amai02b; dng, dbq

Figure 7.11 Summary of the results for axiomatic translation of formulae in local satisfiability calculations in various formulations of axiom S5: The test data consists of formulae 1-117.

The S5 formulations examined are listed below. Formulations for which counter-examples were identified (and hence are not complete) are marked with a strike-through notation.

- KT4B: T_c4_B T4B T_o4_B T_B4_B B_o4_B T_B4_B 4_oT_oB 4_oB_oT
- KD4B: D_c4_B D4B D_o4_B D_B4_B B_o4_B D_B4_B 4_oD_oB 4_oB_oD
- KT5: T_c5_o T_o5_o 5_oT
- KT5_B: T_B5_o T_B5_o T_o5_B 5_oB_oT 5_oT_oB B_oT_o5_o B_o5_oT
- KT4B5: T_c4_B5_o 4_BT5_o 4_oT5_B 4_o5_BT 4_o5_oTB 4_oB_oT5_o 4_oB_o5_oT
- B₄4_T5_o B₄5_oT B₅5_T4 B₅4_T B₅T4₅ B₅T5₄ 5₄4_B5_oT 5₄5_oTB 5_oTB₄ 5_oT4_B 5_oB₄5_oT 5_oB_oT4
- T4_B5_o T4₅5_B T5_B4 T5₄5_B TB₄5_o TB₅4
- T_c4₅ T_o4₅ T_o5₄ 4_oT₅5_o 4_o5_oT 5_oT₄ 5_o4_oT
- KD4B5: D_c4_B5_o D_o4_B5_o D_o4₅B D_o5_B4 D_o5₄B D_oB₄5_o D_oB_o5_o4
- 4_B4_B5_o 4_B5_oD 4_B5_oD 4_B5_oD 4_B5_oD 4_B5_oD
- B₄4_D5_o B₄5_oD B₅5_D4 B₅5_o4D B₅5_o4D B₅5_o4D B₅5_o4D
- 5₄4_B5_o 5₄5_oD B₅5_D4 B₅5_o4D B₅5_o4D B₅5_o4D
- KDB5: D_c5_B D_o5_B B_o5_B 5_oD_B 5_oB_oD B₅5_B B₅5_B

Counter-examples were identified for some formulations for the following formulae. Formulae 1-117 were screened:

- T; 4; 5; 4²; 4³; aiml02_prop3iii; amai02; amai02b; bp, dddnp, g; bq, ddddndq; bq, ddddndq; ddddndq, b(q, g); ddddndq, bq; ddddndq, bq; ddddndq, bq; demri1; demri2; dng, dbq

Mean execution times for local satisfiability over all the problems 1-117 for the various formulations of S5 that were tested are shown below. The formulations that have a strike-through notation are known to be incomplete (since counter-examples were found). The data is sorted by execution time to illustrate the faster formulations. In order to avoid bias, the data *includes* time points for problems that failed to produce a result before the SPASS time out was reached (these are given a 200 seconds execution time; the real execution time must be greater than 200 seconds), and hence the mean execution times are often under-estimates of the true values.

S5 Formulation	Mean execution time (msec)	S5 Formulation	Mean execution time (msec)	S5 Formulation	Mean execution time (msec)
T4B	33	5oDoB	4120	DoBo5o4	7147
4oTB	70	4oBoD	4125	Bo4o5oT	7156
T4oB	76	4oT5o	4193	Bo4oT5o	7443
D4B	99	T4o5o	4218	TBo4o5o	7471
BoT4	109	4o5oTB	4240	Bo5oB	7643
TBo4	131	Bo4oB	4263	4o5oBoD	7671
5oT4	217	5oBoD	4678	BoT4o5o	7693
5oT	252	4o5oDoB	4685	5o4oBoD	7819
T5o4	266	T5oBo4	4749	BoBo5e	7871
T5e	401	T5o4oB	5086	4oDo5oD	7894
5oTB	779	4o5oBoT	5139	Do5oBo4	8236
Bo4oT	1099	4oBo5oT	5343	4oBo5oD	8499
4oBoT	1094	5oDo4oB	5352	Do5o4oB	8877
T5oB	1220	4oT5oB	5403	Bo4o5oD	9670
5o4oT	1687	T4o5oB	5478	BoBo5e	10206
DoBo4	2603	5oDoBo4	5676	4oBoDo5o	11411
BoT5o4	2701	5o4oDoB	5680	4oDoBo5o	11840
Bo5oT4	2725	Bo5oDo4	5737	Bo4oDo5o	12507
5oBoT	2760	BoDo5o4	5964	Do4o5oB	14006
Bo5oT	2831	Do5oB	5967	BoDo4o5o	18143
5oBoT4	2915	5oBo4oT	6199	Do4oBo5o	18825
BoBo4	2992	TBo5e	6231	DoBo4o5o	22553
Do4oB	3009	Bo5o4oT	6293	Tc5c	64338
BoT5o	3124	4oBoT5o	6308	TcBc5c	72051
TBo5o4	3145	5oBoDo4	6347	Tc4cBc5c	78390
5o4oTB	3575	5o4oBoT	6645	Tc4cB	82768
5oT4oB	3593	4oTBo5o	6816	Tc4c5c	92894
4oDoB	3910	T4oBo5o	6859	De4cB	104739
5oTBo4	4060	Bo5o4oD	6903	De4cBc5	105320
4o5oT	4087	5oBo4oD	6958	De5cBc	106087

Figure 7.12: Execution times for the eml-translation to first-order logic component of the axiomatic translation for all the examples in this study:

Execution times in SPASS are reported in the output file. These are approximately equal to the CPU execution times plus the disk access times for input/output. The total execution time, and the execution times of various modules are reported (for example, input / translation from EML to FOL / inferences / backtracking / reduction). Over all the test examples, with all unique cases of applied axioms and axiom combinations considered (25642 cases) the execution times for the EML to FOL translation ranges up to a maximum of 1.06 seconds, with the following statistics.

< 0.01 seconds	56.4 %
0.01-0.1 seconds	41 %
0.1-1.06 seconds	2.6 %

Figure 7.13. Comparing execution times of classical and axiomatic schema translations of modal axioms in local satisfiability calculations: The table shows the number of test examples (from the formulae 1-119 in the test set) for which execution times of the local satisfiability calculation in classical translation is, the same as, greater than, and lower than the execution time of the axiomatic schema translation. The data is also subdivided according to whether the outcome of the calculation in SPASS was Proof (unsatisfiable) or Completion (satisfiable). The mean ratios of execution times for axiomatic schema and classical translations are given. The numbers of test cases where formulae were solved in the axiomatic translation, but not in the classical translation, within the cutoff time of 200 seconds, is also reported. Table cells in which there is no data available, either because the calculation ran out of time for the all cases of the classical translation, or in all cases neither calculation produced a result, are empty.

Execution time classical = axiomatic	Execution time classical > axiomatic	Execution time classical < axiomatic	Solved in axiomatic, but not in classical

Axiom	Complete		Total		Proof		Complete		Total		Proof		Complete		Total		Proof				
	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio	No.	Mean ratio			
T	3		4		1		-		1	0.40	1	0.40	101	2.4	114	2.5	13	2.7	-	-	-
B	34		44		10	0.60	70	0.61	10	0.68	4	1.1	5	1.4	1	3	-	-	-	-	-
D	14		19		5	0.20	2	0.20	-	-	-	95	2.8	98	2.8	3	2.2	-	-	-	-
4	-		7		7	-	18	0.21	18	0.21	-	-	-	-	-	-	-	-	93	94	1
5	-		13		13	-	28	0.26	28	0.26	-	-	7	1.5	7	1.5	7	1.5	66	71	5
Alt ₁	-		3		3	-	14	0.49	14	0.49	-	-	1	2	1	2	100	101	1	1	1
4 ²	-		2		2	-	10	0.11	10	0.11	-	-	-	-	-	-	-	-	106	107	1
4 ³	-		2		2	-	4	0.11	4	0.11	-	-	-	-	-	-	-	-	84	88	4
5 ²	-		1		1	-	12	0.031	12	0.030	-	-	2	2.3	2	2.3	47	59	12	12	12
5 ³	-		1		1	-	2	0.0011	2	0.0011	-	-	2	3.6	2	3.6	6	18	12	12	12
Alt _{1,1,1}	-		1		1	-	7	0.055	7	0.055	-	-	2	2	2	2	78	81	3	3	3
Alt _{1,1,1,1}	-		1		1	-	5	0.090	5	0.090	-	-	2	2	1	2	55	78	23	23	23
Alt _{1,1,2}	-		1		1	-	2	0.15	2	0.15	-	-	1	2	1	2	43	63	20	20	20
Alt _{1,2,2}	-		1		1	-	1	0.47	1	0.47	-	-	1	2	1	2	19	24	5	5	5
T4	-		13		13	-	22	0.10	22	0.10	-	-	1	2	1	2	77	83	6	6	6
TB	33		52		19	0.58	61	0.58	11	0.58	6	1.3	6	1.3	-	-	-	-	-	-	-
DB	30		39		9	-	-	-	-	-	64	2.8	80	2.8	16	2.8	-	-	-	-	-
D4	-		5		5	-	22	0.031	22	0.031	14	4.7	15	4.6	1	2	72	77	5	5	5
4 _B	-		11		11	-	47	0.25	47	0.25	-	-	11	2.1	11	2.1	32	50	18	18	18
5 _B	-		17		17	-	40	0.33	40	0.33	-	-	14	2.1	14	2.1	32	47	15	15	15
5 _T	-		14		14	-	58	0.36	58	0.36	-	-	5	1.7	5	1.7	26	42	16	16	16
T4B	-		17		17	-	38	0.17	38	0.17	-	-	13	1.9	13	1.9	26	51	25	25	25
D _{4,B}	-		2		2	-	33	0.15	33	0.15	-	-	22	2.8	22	2.8	23	59	36	36	36

Figure 7.14: Scott-Lemmon translation (correspondence properties) for selected modal-axioms. The result given is for translation of the formula 5 ($\neg(\rightarrow(\neg\Box\neg\Box p, \Box p))$) in each axiom (using 500 seconds execution time; C = completion (Satisfiable); P = Proof (Unsatisfiable); X = ran out of execution time) in local satisfiability calculations. The SLF-index is the value of the exponents h, i, j, k in $\Box^h\Box^i p \rightarrow \Box^j\Box^k p$. Some of the axioms correspond to well-known modal axioms that are described elsewhere in this study, and are named in the second column. The actual formulae generated by the eml-module are reported.

** Several formulations give rise to the same axiom under the Scott-Lemmon algorithm – for example those marked

SLF index	Axiom Name	Modal Axiom	Correspondence Property	Term created within extended SPASS	Result
0000	None	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	TRUE		C
1000		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow (W=V))$		P
0100	T	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] ((W=\neg V) \rightarrow R_c(W, V))$		C
0010		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow (W=V))$		P
0001		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] ((W=\neg V) \rightarrow R_c(W, V))$		C
1100		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow R_c(W, V))$ **		C
1010	Alt ₁	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, U] ((R_c(U, V) \wedge R_c(U, W)) \rightarrow (W=V))$		X
1001		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow R_c(W, V))$ **		C
0110		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow R_c(W, V))$ **		C
0101	D	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] ((W=\neg V) \rightarrow \exists [X] (R_c(V, X) \wedge R_c(W, X)))$		C
0011	B	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow R_c(W, V))$ **		C
0111		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow \exists [X] (R_c(V, X) \wedge R_c(W, X)))$		C
1101		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V] (R_c(V, W) \rightarrow \exists [X] (R_c(V, X) \wedge R_c(W, X)))$		C
1011	5	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, U] ((R_c(U, V) \wedge R_c(U, W)) \rightarrow R_c(W, V))$		P
1110		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, U] ((R_c(U, V) \wedge R_c(U, W)) \rightarrow R_c(W, V))$		P
1111	G	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, U] ((R_c(U, V) \wedge R_c(U, W)) \rightarrow \exists [X] (R_c(V, X) \wedge R_c(W, X)))$		X
0120	4	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, Y] ((R_c(Y, W) \wedge R_c(V, Y)) \rightarrow R_c(W, V))$		X
0210	DEN	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Y, W, V] (R_c(V, W) \rightarrow (R_c(Y, V) \wedge R_c(W, Y)))$		P
1002		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Y, W, V] (R_c(V, W) \rightarrow (R_c(Y, V) \wedge R_c(W, Y)))$		P
1120	DBBB	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [V, W, Y] ((R_c(U, V) \wedge R_c(Y, W) \wedge R_c(U, Y)) \rightarrow R_c(W, V))$		X
2100	B ²	$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V, Y] ((R_c(Y, V) \wedge R_c(W, Y)) \rightarrow R_c(W, V))$		X
0002		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Y, W, V] ((W=\neg V) \rightarrow (R_c(Y, V) \wedge R_c(W, Y)))$		P
0020		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Y, W, V] ((R_c(Y, W) \wedge R_c(V, Y)) \rightarrow (W=\neg V))$		X
2200		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Z, W, V, Y] ((R_c(Y, V) \wedge R_c(W, Y)) \rightarrow (R_c(Z, V) \wedge R_c(W, Z)))$		C
2220		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, V, Z, U, W, Y] (((R_c(Z, V) \wedge R_c(U, Z)) \wedge (R_c(Y, W) \wedge R_c(U, Y))) \rightarrow (R_c(X, V) \wedge R_c(W, X)))$		C
2020		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Z, U, W, V, Y] ((R_c(Z, V) \wedge R_c(U, Z)) \wedge (R_c(Y, W) \wedge R_c(U, Y)) \rightarrow (W=\neg V))$		X
0202		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Z, Y, W, V] ((W=\neg V) \rightarrow \exists [X] ((R_c(Z, X) \wedge R_c(V, Z)) \wedge (R_c(Y, X) \wedge R_c(W, Y))))$		P
2022		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, V, Z, U, W, Y] (((R_c(Z, V) \wedge R_c(U, Z)) \wedge (R_c(Y, W) \wedge R_c(U, Y))) \rightarrow (R_c(X, V) \wedge R_c(W, X)))$		C
2202		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, Z, W, V, Y] ((R_c(Y, V) \wedge R_c(W, Y)) \rightarrow \exists [X] ((R_c(X, X) \wedge R_c(V, X)) \wedge (R_c(Z, X) \wedge R_c(W, Z))))$		C
2000		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [W, V, Y] ((R_c(Y, V) \wedge R_c(W, Y)) \rightarrow (W=\neg V))$		X
2222		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, X, V, Z, U, W, Y] (((R_c(Z, V) \wedge R_c(U, Z)) \wedge (R_c(Y, W) \wedge R_c(U, Y))) \rightarrow \exists [X] ((R_c(X, X) \wedge R_c(V, X)) \wedge (R_c(X, X) \wedge R_c(W, X))))$		C
0022		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [Z, V, W, Y] ((R_c(Y, W) \wedge R_c(V, Y)) \rightarrow (R_c(Z, V) \wedge R_c(W, Z)))$		C
0222		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, Z, V, W, Y] ((R_c(Y, W) \wedge R_c(V, Y)) \rightarrow \exists [X] ((R_c(X, X) \wedge R_c(V, X)) \wedge (R_c(Z, X) \wedge R_c(W, Z))))$		C
3333		$\Box^h\Box^i p \rightarrow \Box^j\Box^k p$	$\forall [X, X, X, X, V, X, X, U, W, Z, Y] (((R_c(X, V) \wedge R_c(X, X) \wedge R_c(U, X)) \wedge (R_c(Z, W) \wedge R_c(Y, Z) \wedge R_c(U, Y))) \rightarrow \exists [X] ((R_c(X, X) \wedge R_c(V, X)) \wedge (R_c(Z, X) \wedge R_c(W, Z))))$		C

nq, ddddbq	P	P	P	P	P	P	P	P	P
p, bbndp, ddq	P	P	P	P	P	P	P	P	P
(db)^1p, p	P	P	P	P	P	P	P	P	P
(db)^1p, q	X	C	C	X	X	X	X	X	X
(db)^2p, p	P	P	P	P	P	P	P	P	P
(db)^2p, q	X	C	C	X	X	X	X	X	X
(db)^3p, p	P	P	P	P	P	P	P	P	P
(db)^3p, q	X	C	C	X	X	X	X	X	X
(db)^4p, p	P	P	P	P	P	P	P	P	P
(db)^4p, q	X	C	C	X	X	X	X	X	X
(db)^5p, p	P	P	P	P	P	P	P	P	P
(db)^5p, q	X	C	C	X	X	X	X	X	X
(db)^6p, p	P	P	P	P	P	P	P	P	P
(db)^6p, q	X	C	C	X	X	X	X	X	X
(db)^7p, p	P	P	P	P	P	P	P	P	P
(db)^7p, q	X	C	C	X	X	X	X	X	X
(db)^8p, p	P	P	P	P	P	P	P	P	P
(db)^8p, q	X	C	C	X	X	X	X	X	X
(db)^9p, p	P	P	P	P	P	P	P	P	P
(db)^9p, q	X	C	C	X	X	X	X	X	X
(db)^10p, p	P	P	P	P	P	P	P	P	P
(db)^10p, q	X	C	C	X	X	X	X	X	X
aim102_prop3i	P	P	C	P	P	P	P	P	P
aim102_prop3ii	P	P	P	X	P	P	P	P	P
aim102_prop3iii	P	P	P	P	P	P	P	P	P
amai02	P	P	P	P	P	P	P	P	P
amai02b	P	P	P	P	P	P	P	P	P
demr1	P	P	C	P	P	P	P	P	P
demr2	X	P	P	X	P	P	P	P	P
demr3	P	P	P	P	P	P	P	P	P
demr5	X	C	C	X	C	C	P	P	X
demr6	X	C	C	X	X	X	X	X	X
demr7	P	P	C	X	P	P	P	P	P
demr8	X	C	C	X	P	P	P	P	P
demr9	X	C	C	X	X	X	X	X	X
CR	P	P	C	X	P	P	P	P	P
demr14	P	P	P	X	P	P	P	P	P

T_{4B_c}	119(0)	20	27439	330	95	20	20413	80	24	20	57476	330
D_{cB}	119(0)	20	27427	210	71	10	17411	60	48	25	40337	210
D_{c4}	119(0)	20	25428	200	94	20	20413	90	25	20	47451	200
T_c	119(0)	20	21417	100	65	10	16411	80	54	20	27421	100
D_c	119(0)	20	21417	100	61	10	16412	80	58	20	26420	100
B_c	119(0)	20	26427	170	66	10	1649	50	53	20	37435	170
4_c	69(50)	10	23241415	11740	66	15	22541442	11740	3	10	3904658	1150
5_c	90(29)	20	1314749	6920	87	20	564198	1390	3	10	23133989	6920
Alt_{1c}	98(21)	20	19412	80	91	20	19412	80	7	10	1345	20
4_c²	66(53)	10	23241621	13190	64	15	23941646	13190	2	10	1040	10
4_c³	67(52)	20	73144261	34260	65	20	75344325	34260	2	10	1040	10
5_c²	77(42)	20	8417429312	170170	75	20	8642429673	170170	2	10	1040	10
5_c³	60(59)	10	3734425708	198810	58	15	3862426145	198810	2	10	1040	10
Alt_{1c}^{1,1}	91(28)	20	3443416657	145120	84	20	3729417314	145120	7	10	1448	30
Alt_{1c}^{2,1}	93(26)	20	5252419246	124790	86	20	5678419962	124790	7	10	23422	60
Alt_{1c}^{2,2}	93(26)	20	3302410813	60930	86	20	3570411207	60930	7	10	17413	40
Alt_{1c}^{2,2}	78(41)	20	20941185	10220	71	20	22841241	10220	7	10	19416	50
T_{4c}	74(45)	10	31342238	19280	74	10	31342238	19280	0	-	-	0
T_{Bc}	119(0)	20	28431	190	75	10	19423	190	44	30	43437	150
D_{Bc}	119(0)	20	26428	210	71	10	17411	60	48	25	40437	210
D_{4c}	77(42)	20	175346657	48170	70	10	191846965	48170	7	90	100457	190
4_{Bc}	86(33)	20	88446940	64410	83	20	90647065	64410	3	10	270450	790
5_{Bc}	88(31)	20	41541474	10040	85	20	41941498	10040	3	10	2904485	850
T_{5c}	95(24)	20	1194416	3590	95	20	1194416	3590	0	-	-	-
T_{4c}B_c	90(29)	20	2910419652	170350	90	20	2910419652	170350	0	-	-	-
D_{4c}B_c	86(33)	20	81642547	18380	86	20	81642547	18380	0	-	-	-

Figure 7.25 Global satisfiability results from the SPASS resolution prover for the test set of target formulae in each of the listed axiom combinations. The data is taken from the results for test problems 1-119. Problems are deemed to have failed if they were not solved before running out of time at 200 seconds execution time. Execution time statistics are listed for the solved problems. The statistics for all the problems, regardless of outcome, are all listed under the heading *Total*. This set of solved problems is sub-divided by the outcome, either Completion in SPASS (satisfiable), or Proof in SPASS (unsatisfiable), in the other two sections. Where there is no data (no problems solved) the table is blank. The median, arithmetic mean ± standard deviation, and maximum values are recorded.

Axiom	Total set of problems				Problem outcome = Proof				Problem outcome = Completion			
	No. problems solved (failed)	Execution time in msec			No. problems solved	Execution time in msec			No. problems solved	Execution time in msec		
		Median	Mean ±SD	Max		Median	Mean ±SD	Max		Median	Mean ±SD	Max
K	119(0)	20	21±20	140	56	10	16410	50	63	20	25426	140
T	119(0)	30	34±24	150	65	20	25415	60	54	40	44428	150
B	119(0)	20	22±20	170	66	10	18411	60	53	20	28426	170
D	119(0)	20	54±108	820	61	20	26428	190	58	30	844146	820
4	119(0)	20	22±20	130	88	20	19410	60	31	20	33434	130
5	119(0)	20	45±86	870	88	20	33434	280	31	30	784155	870
Alt₁	119(0)	20	54±107	800	92	20	32435	230	27	30	1284201	800
4²	119(0)	20	47±126	1090	87	20	22418	140	32	20	1154230	1090
4³	118(1)	20	78944907	47970	88	20	454170	1610	30	20	297049510	47970
5²	118(1)	40	27541275	13270	87	40	804126	790	31	80	82342424	13270
5³	116(3)	70	232247979	47990	88	60	72842841	21780	28	720	7332414520	47990
Alt₁^{1,1}	114(5)	30	2118412932	123380	91	30	45471	570	23	30	10323427763	12338
Alt₁^{2,1}	115(4)	30	2780415982	145230	93	30	89848011	77300	22	55	10733431992	14523
Alt₁^{2,2}	116(3)	30	2595417905	187220	92	30	32741639	13800	24	100	11291438638	18722
Alt₁^{2,2}	104(15)	30	1986411239	89390	91	30	2125411979	89390	13	20	101842627	8910
T4	119(0)	20	24±25	190	95	20	19410	60	24	20	44447	190
TB	119(0)	20	23±20	150	75	10	18411	60	44	20	32427	150
DB	119(0)	20	51±82	600	71	20	26421	110	48	40	884117	600
D4	119(0)	20	50±91	680	94	20	31431	230	25	30	1224174	680
4_B	119(0)	20	47±96	940	89	20	33435	220	30	20	894177	940
5_B	119(0)	30	594223	2440	89	30	37431	170	30	30	1274439	2440
5_T	119(0)	20	40±65	670	95	20	30423	130	24	30	794134	670
T4_B	119(0)	20	44±70	600	95	20	31428	160	24	35	954137	600
Do4_B	119(0)	50	151±526	5620	95	50	844105	570	24	70	41841131	5620